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ABSTRACT

This volume is the twenty-seventh in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by third-grade teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of four groups of lessons. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. This unit reviews concepts related to multiplication which were introduced in earlier units, then expands these concepts to include multiplication by zero, the use of placeholders in multiplicative problems, multiplication of more than two factors, and the use of the vertical algorithm. Work with partitioning of arrays, using Cartesian products, and solving word problems is included. (SD)

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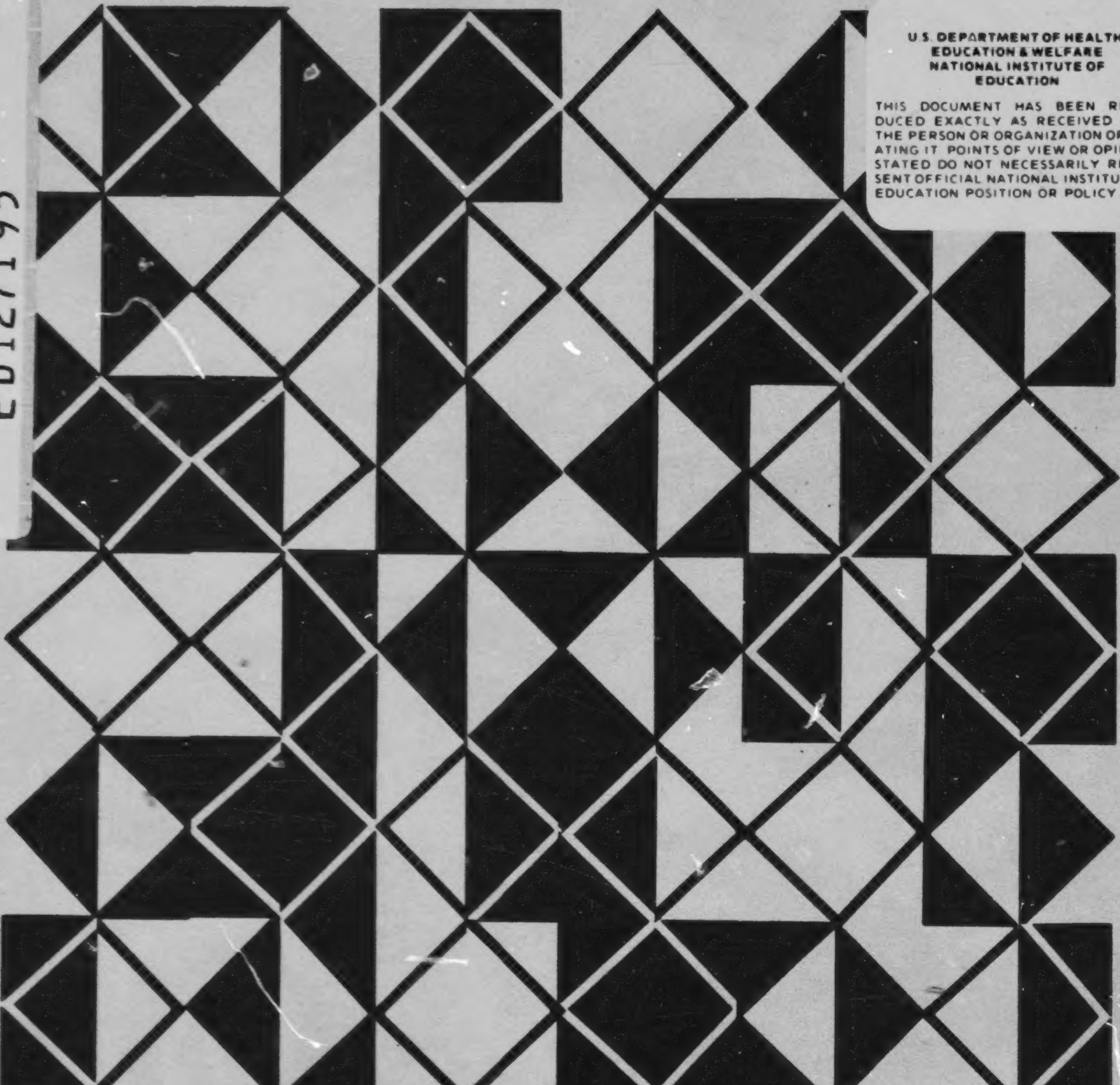

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NUMBERS AND THEIR PROPERTIES

UNIT 27

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Second Printing, 1971

NUMBERS AND THEIR PROPERTIES

This unit was developed by MINNEMAST on the basis of the experiences of teachers who used an earlier trial version.

ROY W. MUCKEY

PAUL REDIN

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The cover design is used by permission of Sonia D. Forseth. (Copyright pending.) The design is a matrix illustrating multiplication in mod seven. There are six elements in the design. These six elements are shown on the top row and left column. The results of multiplying the elements are shown in the cells of the matrix. The same matrix with numerals rather than designs is shown below.

1	2	3	4	5	6
2	2 · 2 4	2 · 3 6	2 · 4 1	2 · 5 3	2 · 6 5
3	3 · 2 6	3 · 3 2	3 · 4 5	3 · 5 1	3 · 6 4
4	4 · 2 1	4 · 3 5	4 · 4 2	4 · 5 6	4 · 6 3
5	5 · 2 3	5 · 3 1	5 · 4 6	5 · 5 4	5 · 6 2
6	6 · 2 5	6 · 3 4	6 · 4 3	6 · 5 2	6 · 6 1

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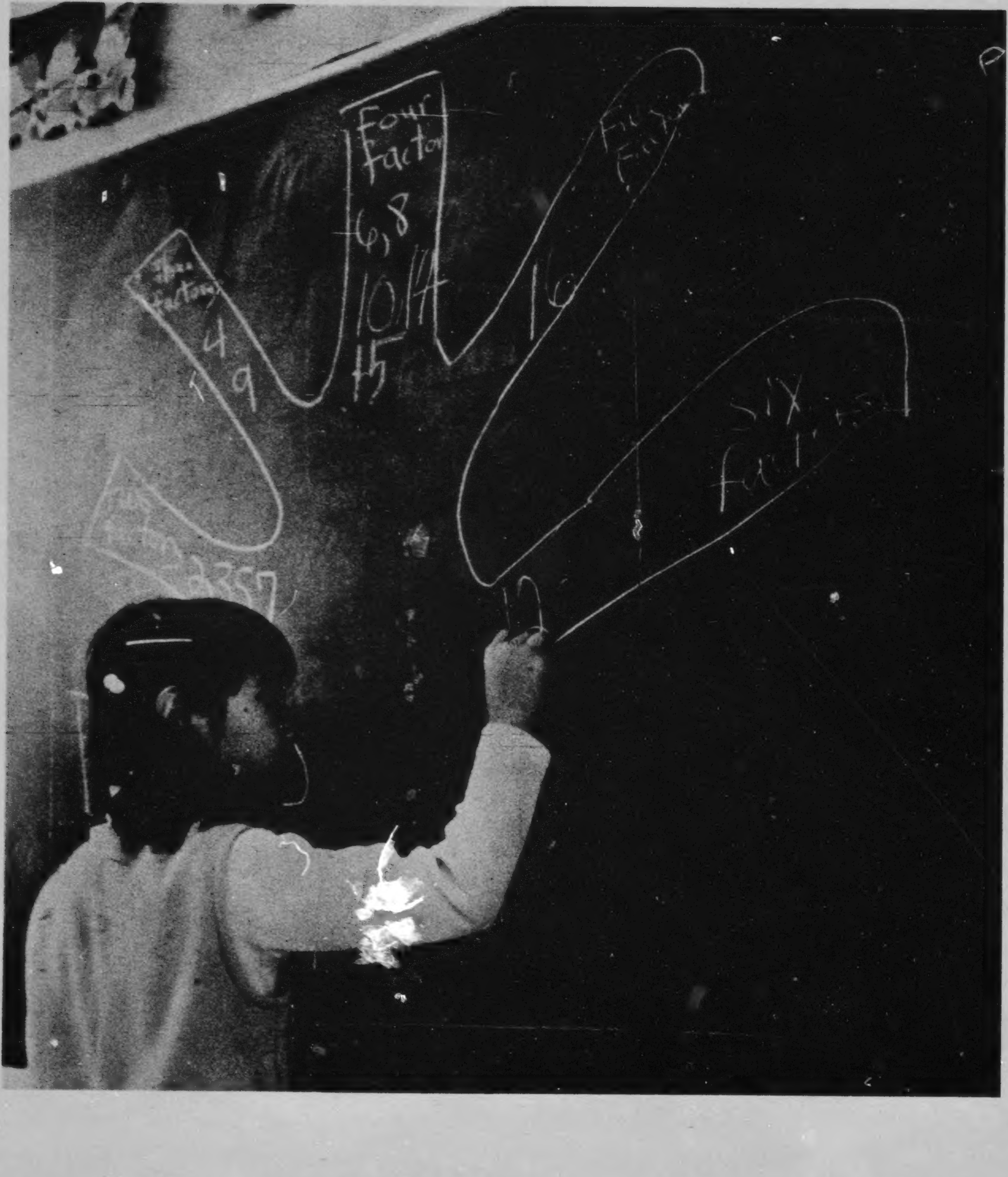
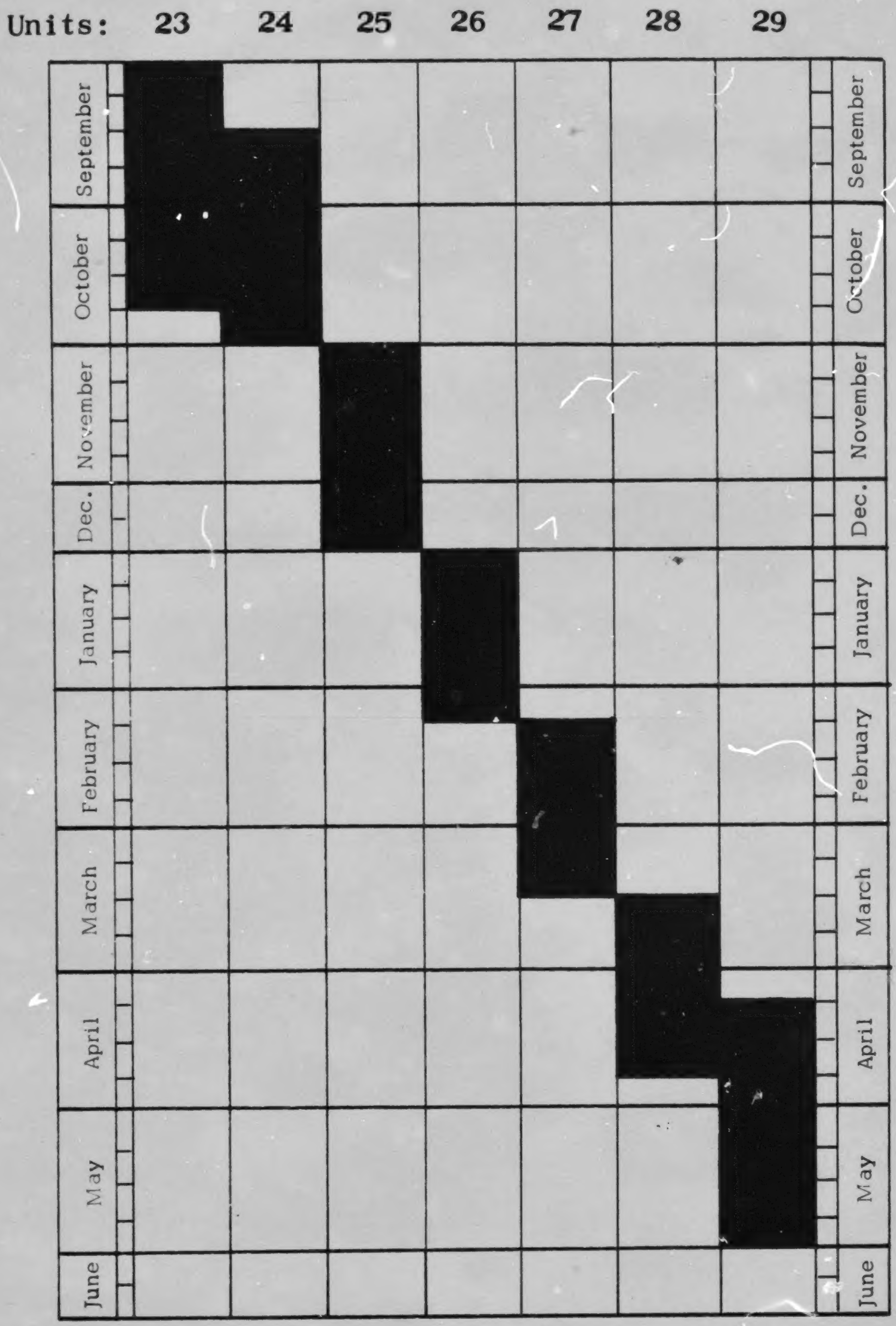

Four Factors
6, 8
10, 14
15
Three Factors
4
9
Five Factors
16
Six Factors
Two Factors
2357

Suggested Teaching Schedule for MINNEMAST Third Grade Units



Complete List of Materials for Unit 27

(Numbers based on class size of 30.)

total number required to teach unit	item	lessons in which item is used
30	** Student Manuals	
10	* spinner discs	3
5	* spinner bases	3
5	* spinner arrows	3
30	* rulers	6
30	pairs of scissors	6,8,10
30	** clock protractors from Unit 26	6
30	* small brass fasteners	8
15	* pair of dice numbered 0-5 and 4-9	10
30 ea.	* red, blue, purple and pink crayons	18
	colored chalk	18,19

for the balance beam job booklet:

15	* meter sticks
15	* wooden blocks
75	* jumbo paper clips of each color: red, black, silver
15	* straightened paper clips
	masking tape

---

* kit items as well as

** printed materials available from Minnemath Center,
720 Washington Avenue S.E., Minneapolis, Minnesota 55455

*** available from The Judy Company
310 North Second Street, Minneapolis, Minnesota 55401

INTRODUCTION

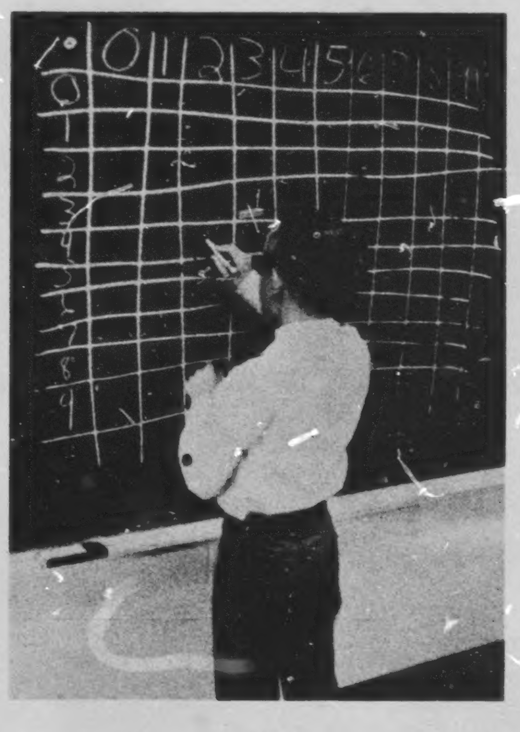
This unit concentrates almost exclusively on multiplication. Included within the unit are many opportunities for independent and group work. The unit is intended to be semi-self-instructional with much of the material developed through worksheets, group work or class discussion.

Section 1 provides more practice with the basic facts while examining the multiplication tables to discover their properties. The multiplication properties of 0 and 1 and symmetry (commutativity) are some of the patterns discussed.

Section 2 uses arrays as an embodiment of multiplication and again the properties are examined. In addition, the distributive property is developed through the partitioning of arrays.

Section 3 develops the standard multiplication algorithm by extending the ideas of partitioning arrays. It is suggested that the balance beam job booklet be used in conjunction with this section. The booklet examines the multiplication relationships demonstrated by the balance beam.

Section 4 reviews Cartesian products, especially their relationship to multiplication and experimentation.



NOTES ON TEACHING THIS UNIT

The worksheets in this unit are designed to encourage independent work by the children, but this does not mean that they need no help from you. As the children work independently, you should check the work of individuals and help them with their problems. It is important that as you help the children, you do not give them the answers, but rather guide them to the answers by appropriate questions and comments.

In each lesson the children do several worksheets alone or with partners. An explanation of the concepts on each worksheet is provided in this manual. If several children have trouble with the same concept, you may want to take them aside and work with them as a group. In some lessons extra worksheets are provided for fast workers. The rate of progress through the student manual will depend on the abilities of the children in your class. Encourage the children to work as quickly as they can and not to spend too much time on a worksheet. The unit should be completed in four to six weeks.

When you are ready to start Section 3, you may want to let half of the students begin the job booklet while the other students do the worksheets. While one group of children is doing the job booklet, you can work more closely with the other students on the worksheets of Section 3. These worksheets examine the multiplication algorithm in vertical form. Most likely, the students will need more supervision and explanation from you in this section than they will on the worksheets of other sections. When the first group completes the job booklet, these students can then do the worksheets with your assistance while the other group does the job booklet. You will probably find that the group doing the worksheets finishes before those doing the job booklet. The extra time should be spent practicing the algorithm. You may want to make additional sheets of practice problems to help the students become proficient with the algorithm.

Section 1

Factors, Products and Multiplication Sentences

SECTION 1. FACTORS, PRODUCTS AND MULTIPLICATION SENTENCES

PURPOSE

In this section the students learn:

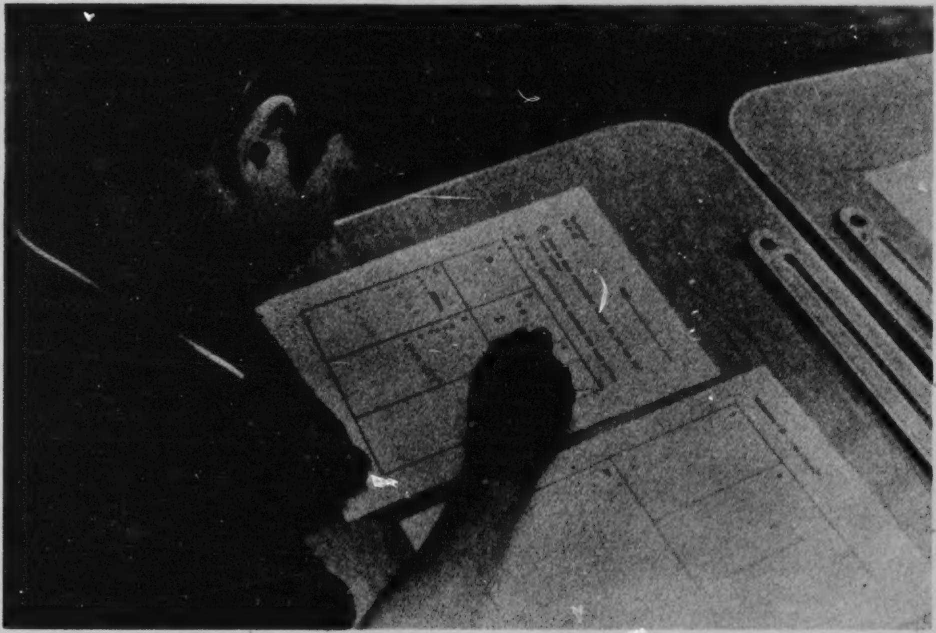
- To work independently -- reading, following instructions and completing the worksheets with a minimum of help from the teacher.
- To identify factors and products in multiplication sentences and to supply missing factors and products in sentences.
- To write multiplication sentences for a given product, to list the factors for a given product, to illustrate multiplication sentences graphically with arrays, and to describe arrays with multiplication sentences.
- To apply the properties of multiplication by 0, 1, 5 and 9 to multiplication problems where those numbers appear as factors.

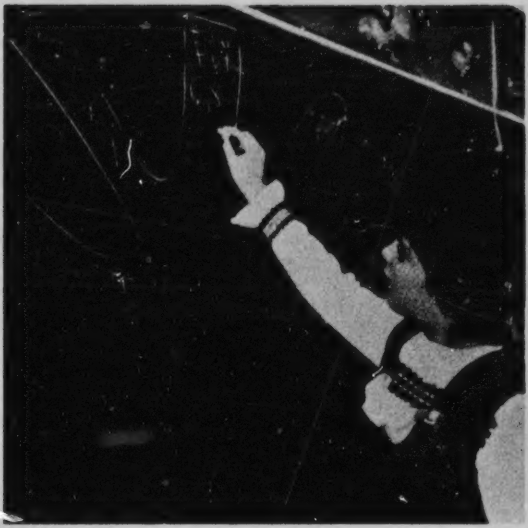
COMMENTARY

In this unit the students will be working independently or with partners on many of the worksheets. Not every student will complete all worksheets or all problems on a worksheet. But, all students should be allowed to work independently and be encouraged to complete as much as they can of each worksheet with a minimum of help from you.

Several lessons include extra worksheets for fast workers. Slower children may do these worksheets at home or in other free time if they wish. Some lessons also include worksheets for class discussion. All children should complete these as a group. The worksheets have been divided to approximate one day's work per lesson. Feel free to adjust this schedule to the abilities of your students. As with most units, this one should be completed within four to six weeks.

In Lesson 1 the students complete several worksheets independently. They identify factors and products in multiplication sentences, and fill in missing parts of other sentences. They draw arrays and write multiplication sentences for those arrays. Then the students write multiplication sentences for given products without drawing arrays.





Lesson 2 continues the work with writing multiplication sentences for products. Then the students isolate the factors for a given product. For example, for the product 16, the factors are 1, 16, 2, 8 and 4. The students complete several worksheets independently and then complete a few more as a class discussion. This discussion reviews the concepts presented on the worksheets of the first two lessons.

Lesson 3 includes two mathematical games that provide practice with the basic multiplication facts and with addition and subtraction. The different versions of each game also provide practice with computational skills. The students should be encouraged to play these games for the remainder of the school year, and especially during this unit when they have completed the independent work of each lesson.

In Lesson 4 the students continue their independent work. They study properties of multiplying by 0 and 1. Whenever 0 is a factor, the product is always 0. Whenever 1 is a factor, the product is always the same as the other factor. The students are introduced to the use of multiplication charts where factors are arranged in numerical order along the top and side of the chart. The students work with ordered pairs and arrays that illustrate multiplication, particularly by 0 and 1.

Lesson 5 introduces the use of a letter as a placeholder for a number in an arithmetic problem. First the students use letters as placeholders in addition problems, and then in multiplication problems. The students do several worksheets independently, and then complete three others as part of a class discussion that serves as review and practice of concepts presented on previous worksheets.

Lesson 6 is completed as a group. The students study bilateral symmetry in pictures and in addition and multiplication charts. They measure angles with the clock protractors that were used in Unit 26.

In Lesson 7 the students work independently on worksheets that provide practice with multiplication facts. They learn some properties of multiplication by 5 and 9. Whenever 9 is a factor, either the product is 9 or the sum of the digits in the product totals 9. Whenever 5 is a factor, the product must end in either 0 or 5. Some worksheets include word problems and number lines that serve as models for multiplication. Other worksheets review the similarity between multiplication and repeated addition.

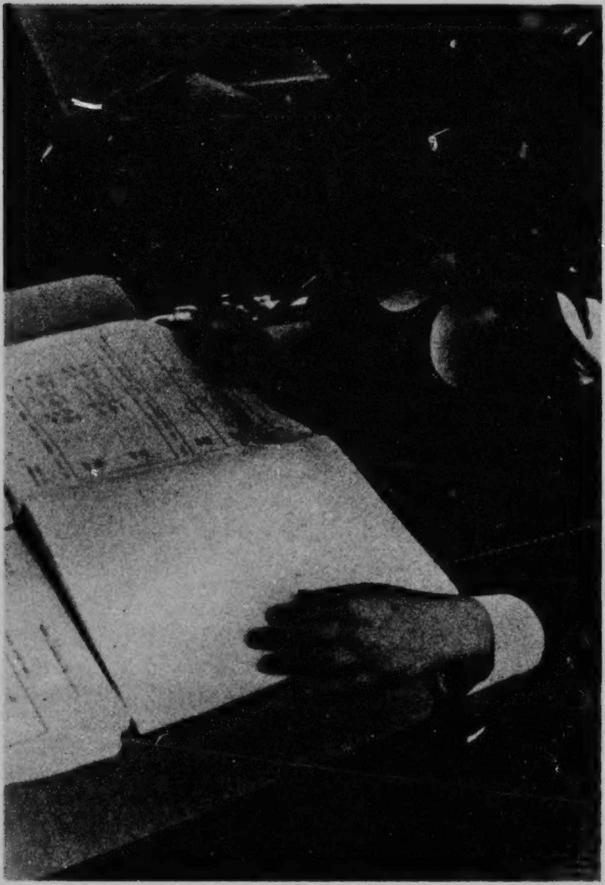
Lesson 1: REVIEWING FACTORS, PRODUCTS, ARRAYS AND MULTIPLICATION SENTENCES

In this lesson the children study the words "factor" and "product." They find missing factors and products. They review rectangular arrays, write multiplication sentences for arrays and draw arrays for sentences. The children draw arrays for the counting numbers 1-10.

MATERIALS

- Worksheets 1-7 (independent)

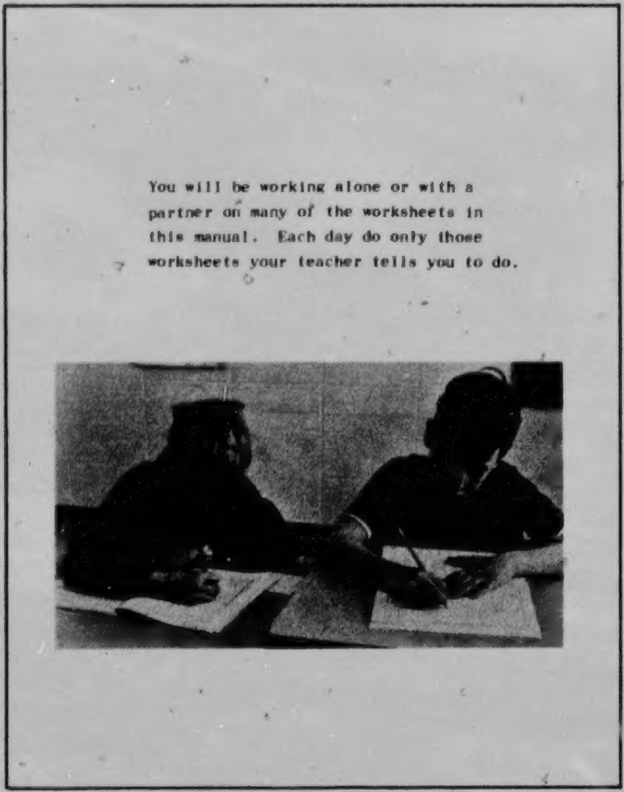
PROCEDURE



In this lesson the children complete Worksheets 1 through 7 independently or with partners during one class period. As the children work, go around the room checking their progress. Do not give away answers, but lead the children to discover their own answers when they have problems. If several children have trouble with the same concept, take those children aside and lead a group discussion that will clear up the problems. The content of each worksheet is explained on the following pages. (Any children who finish these worksheets rapidly, should go on with Worksheets 8 and 9 in the next lesson.)

Explain to the children the working procedure for this unit. In most lessons the children will do a set of worksheets independently or with partners. You may want the children to work alone and only occasionally check their work with a friend. (Several worksheets tell the children to check answers with a friend.) Decide which procedure you will follow and explain this to the children.

Now have the children do Worksheets 1 through 7 independently or with partners for one class period as you go around the room checking their work. If children are having trouble or are not working fast enough, help them with these problems but do not give away the answers.



You will be working alone or with a partner on many of the worksheets in this manual. Each day do only those worksheets your teacher tells you to do.

Worksheets 1 and 2 give practice in using the words "factor" and "product," identifying factors and products, and filling in missing factors and products.

On Worksheet 3 the children study rectangular arrays. They see that a multiplication sentence can describe the array. The first number in the sentence tells the number of rows; the second number tells the number of objects in each row. The children write multiplication sentences to describe arrays.

Worksheet 1
Unit 27

Name ____________________

① Each numeral in a multiplication sentence has a name. The names are: factor and product.

factor factor product

2 · 3 = 6

② Let's look at the product.
The product of each sentence below is circled.

5 · 1 = ⑤ ⑫ = 3 · 4 7 · 9 = (56)

③ Circle the product in each sentence:

3 · 7 = ㉑ 5 · 9 = ㊺ 8 · 6 = ㊽

Caution: ⑱ = 2 · 9 (81) = 9 · 9

In the sentences above, the products are 21, 45, 48, 18, and 81.

④ Fill in the factor [✓ product] (circle one).

5 · 8 = 40 3 · 7 = 21 4 · 8 = 32
2 · 10 = 20 7 · 2 = 14 3 · 6 = 18

Worksheet 2
Unit 27

Name ____________________

① factor factor product

2 · 3 = 6

Which numbers are factors? 2 and 3.

② 5 · 3 = 15. 15 is the product.
5 and 3 are the factors.

③ Circle each factor in these sentences:

④ · ⑦ = 28 ⑨ · ⑧ = 72 ⑥ · ④ = 24

④ In the sentences above the factors are:
4 and 7, 9 and 8, 6 and 4.

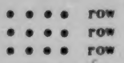
⑤ Fill in the missing factors:

2 · 3 = 6 5 · 2 = 10 14 = 2 · 7

Worksheet 3
Unit 27

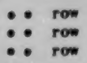
Name ____________________

①



This is an array.
Write the word array here array

② The multiplication sentence 3 · 2 = 6 describes this array.

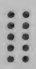


The array has 3 rows and 2 objects in each row.

③ The first number in the multiplication sentence tells the number of rows.

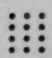
The second number tells the number of objects in each row.

④ Here is an array.



5 · 2 = 10 describes the array.
How many rows are there in the array? 5
How many objects in each row? 2

⑤ Write a sentence for this array:

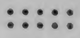


4 · 3 = 12

This array has 4 rows with 3 objects in each row.
The multiplication sentence is 4 · 3 = 12.

Worksheet 3 (Continued)

⑥ Write a sentence to describe this array:

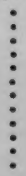


2 · 5 = 10

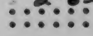
The first number tells the number of rows.
There are 2 rows in the array.
The second number tells the number of objects in each row.
There are 5 objects in each row.

⑦ Write a multiplication sentence for each array.

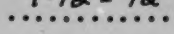
12 · 1 = 12

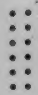


2 · 6 = 12



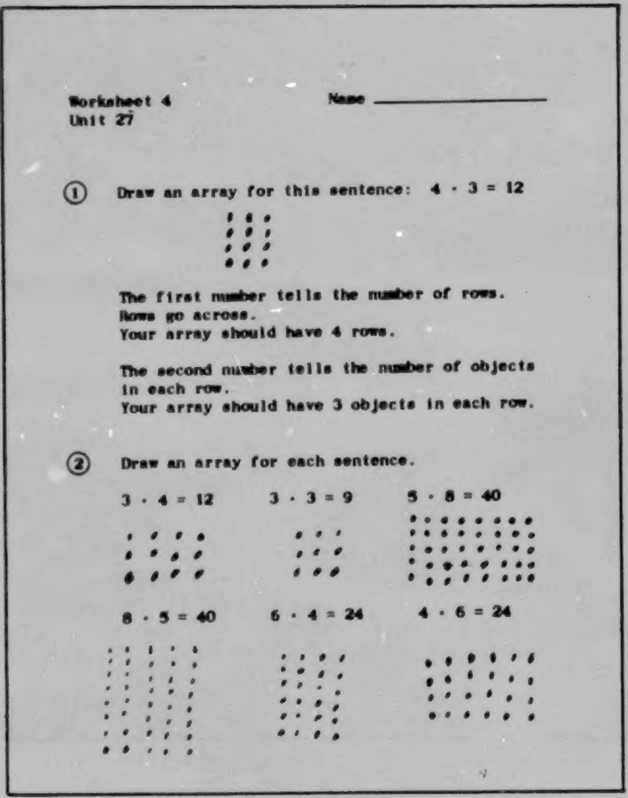
1 · 12 = 12





6 · 2 = 12

⑧ Check your work with a friend.



Worksheet 4
Unit 27

Name ______________

① Draw an array for this sentence: 4 · 3 = 12

The first number tells the number of rows.
Rows go across.
Your array should have 4 rows.

The second number tells the number of objects in each row.
Your array should have 3 objects in each row.

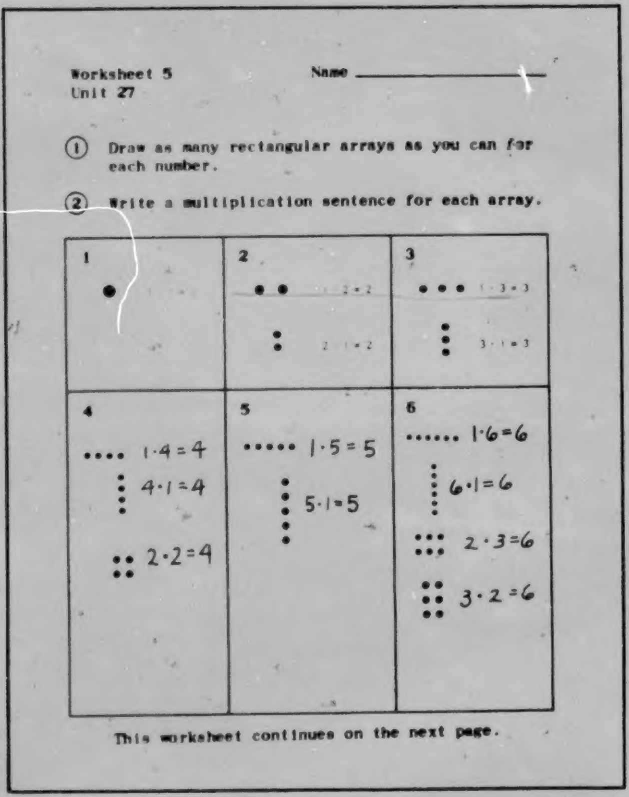
② Draw an array for each sentence.

3 · 4 = 12   3 · 3 = 9   5 · 8 = 40

8 · 5 = 40   6 · 4 = 24   4 · 6 = 24

On Worksheet 4 the children draw arrays for multiplication sentences.

On Worksheet 5 the children make arrays and multiplication sentences for the counting numbers 1-10. For example, they draw all possible arrays that have a total of 9 objects. Then they write a multiplication sentence for each array. The children should save this worksheet for reference when they get to later worksheets that ask them to write all the possible multiplication sentences for a product and to find all the factors for a product.



Worksheet 5
Unit 27

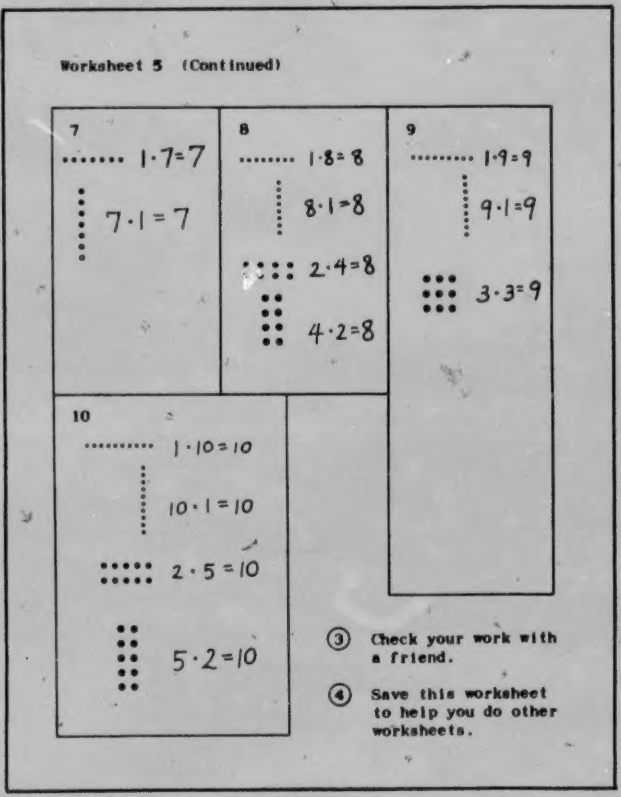
Name ______________

① Draw as many rectangular arrays as you can for each number.

② Write a multiplication sentence for each array.

1 1 · 1 = 1	2 1 · 2 = 2 2 · 1 = 2	3 1 · 3 = 3 3 · 1 = 3
4 1·4=4 4·1=4 2·2=4	5 1·5=5 5·1=5	6 1·6=6 6·1=6 2·3=6 3·2=6

This worksheet continues on the next page.

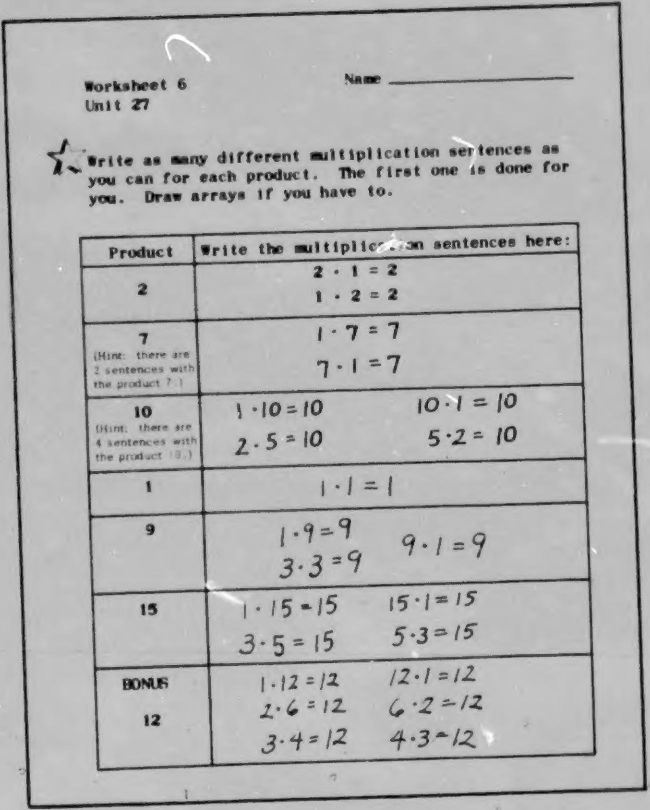


Worksheet 5 (Continued)

7 1·7=7 7·1=7	8 1·8=8 8·1=8 2·4=8 4·2=8	9 1·9=9 9·1=9 3·3=9
10 1·10=10 10·1=10 2·5=10 5·2=10		

③ Check your work with a friend.

④ Save this worksheet to help you do other worksheets.

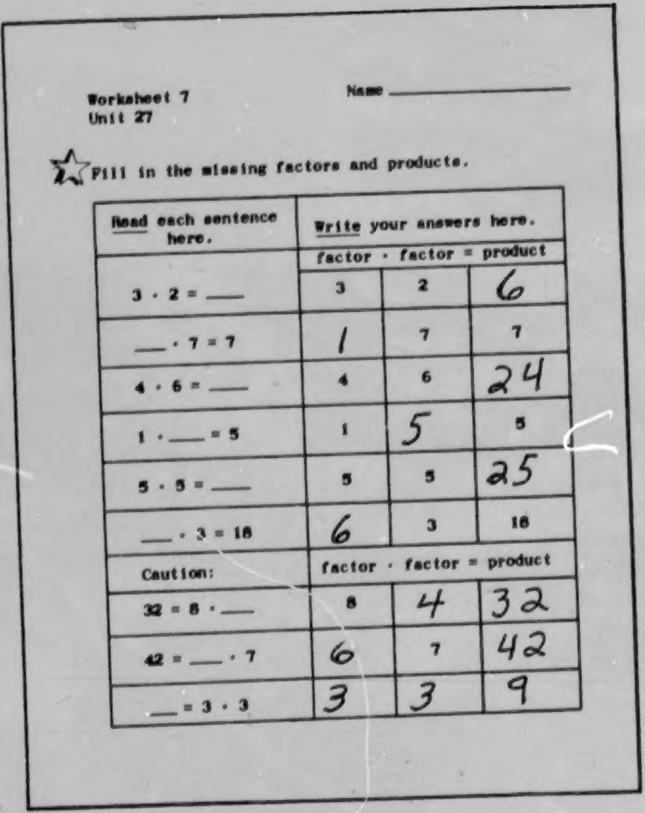


Worksheet 6
Unit 27

Name ____________

Write as many different multiplication sentences as you can for each product. The first one is done for you. Draw arrays if you have to.

Product	Write the multiplication sentences here:
2	2 · 1 = 2 1 · 2 = 2
7 (Hint: there are 2 sentences with the product 7.)	1 · 7 = 7 7 · 1 = 7
10 (Hint: there are 4 sentences with the product 10.)	1 · 10 = 10 10 · 1 = 10 2 · 5 = 10 5 · 2 = 10
1	1 · 1 = 1
9	1 · 9 = 9 9 · 1 = 9 3 · 3 = 9
15	1 · 15 = 15 15 · 1 = 15 3 · 5 = 15 5 · 3 = 15
BONUS 12	1 · 12 = 12 12 · 1 = 12 2 · 6 = 12 6 · 2 = 12 3 · 4 = 12 4 · 3 = 12

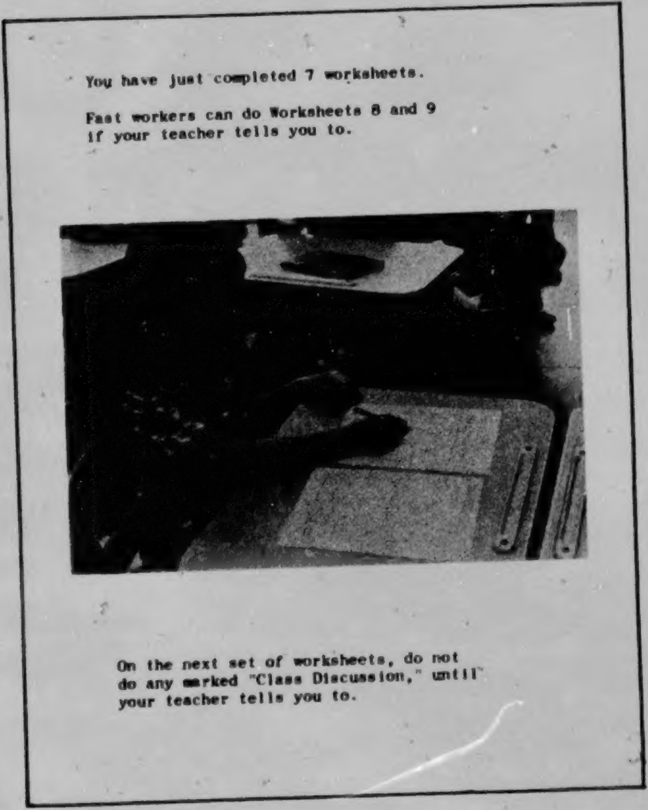


Worksheet 7
Unit 27

Name ____________

Fill in the missing factors and products.

Read each sentence here.	Write your answers here.		
	factor	· factor	= product
3 · 2 = ___	3	2	6
___ · 7 = 7	1	7	7
4 · 6 = ___	4	6	24
1 · ___ = 5	1	5	5
5 · 5 = ___	5	5	25
___ · 3 = 18	6	3	18
Caution:	factor	· factor	= product
32 = 8 · ___	8	4	32
42 = ___ · 7	6	7	42
___ = 3 · 3	3	3	9



You have just completed 7 worksheets.

Fast workers can do Worksheets 8 and 9 if your teacher tells you to.

On the next set of worksheets, do not do any marked "Class Discussion," until your teacher tells you to.

On Worksheet 6 the children are asked to write multiplication sentences for products. They are told to draw arrays if they have to.

Worksheet 7 gives practice in filling in missing factors and products. In the process, the children also practice their multiplication facts. If the children do not know the products for some problems, suggest that they use their multiplication machines from Unit 25, counters, or arrays to find the products.

Lesson 2: WRITING MULTIPLICATION SENTENCES FOR PRODUCTS

The children write multiplication sentences for given products and study the factors for a given product. As a class, they discuss the concepts on the first set of worksheets.

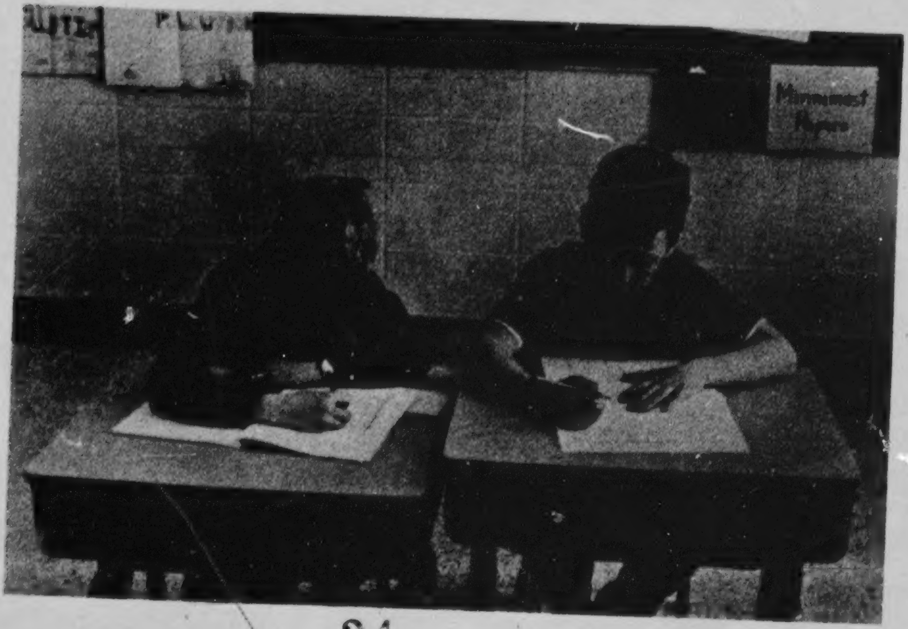
MATERIALS

- Worksheets 8-9 (independent)
- Worksheets 10-11 (fast workers)
- Worksheets 12-14 (discussion)

PROCEDURE

In Activity A the children do Worksheets 8 and 9 independently. Fast workers should complete Worksheets 10 and 11 also. Review with the children the procedure for working independently.

In Activity B, the children discuss Worksheets 12, 13 and 14 as a group. Activities A and B should be completed in one class period. Activity C is an optional commentary about the worksheets.



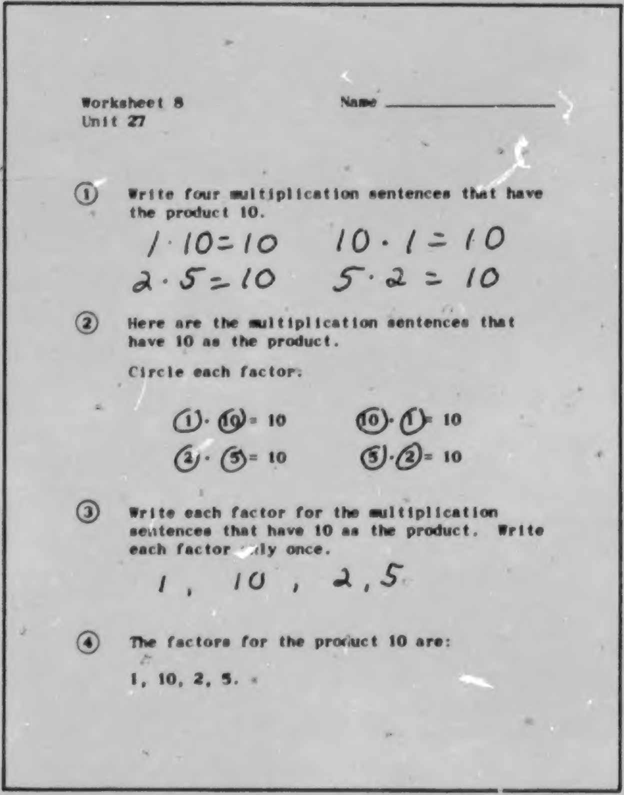
Activity A

Worksheet 8 asks the children to write four multiplication sentences with 10 as a product. The children circle each factor in each sentence, and then write each different factor. They write each factor only once: 1, 10, 2, and 5.

On Worksheet 9 the children write five multiplication sentences for the product 16, and then write the five different factors: 1, 16, 2, 8, and 4. They also write sentences and factors for the product 9.

Worksheets 10 and 11 are for fast workers. On Worksheet 10, the children draw arrays for the counting numbers 11-20, and then write a multiplication sentence to describe each array.

On Worksheet 11 the children write sentences for certain products, and then list the different factors for each product.



Worksheet 8
Unit 27

Name ____________________

① Write four multiplication sentences that have the product 10.

1 · 10 = 10 10 · 1 = 10
2 · 5 = 10 5 · 2 = 10

② Here are the multiplication sentences that have 10 as the product.

Circle each factor.

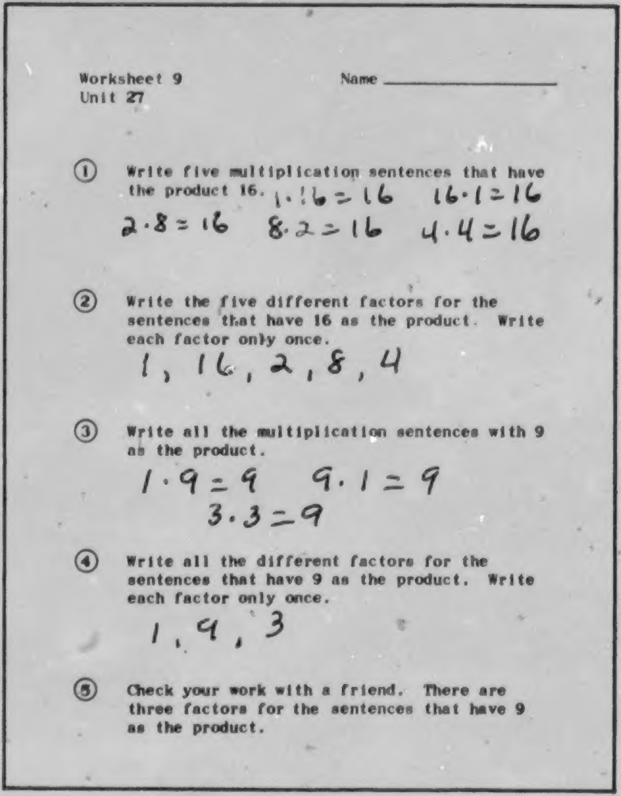
①·⑩= 10 ⑩·①= 10
②·⑤= 10 ⑤·②= 10

③ Write each factor for the multiplication sentences that have 10 as the product. Write each factor only once.

1, 10, 2, 5

④ The factors for the product 10 are:

1, 10, 2, 5.



Worksheet 9
Unit 27

Name ____________________

① Write five multiplication sentences that have the product 16. 1 · 16 = 16 16 · 1 = 16
2 · 8 = 16 8 · 2 = 16 4 · 4 = 16

② Write the five different factors for the sentences that have 16 as the product. Write each factor only once.

1, 16, 2, 8, 4

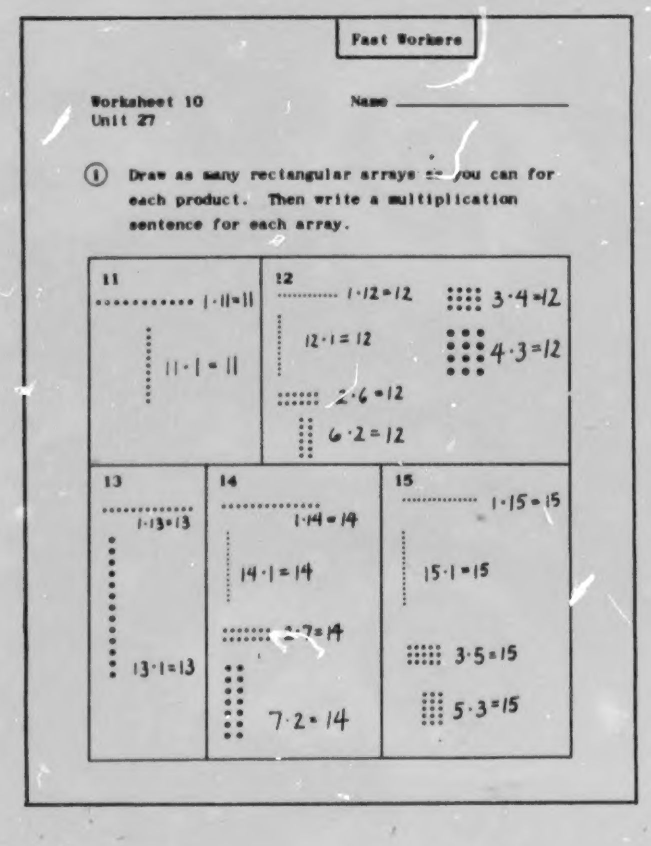
③ Write all the multiplication sentences with 9 as the product.

1 · 9 = 9 9 · 1 = 9
3 · 3 = 9

④ Write all the different factors for the sentences that have 9 as the product. Write each factor only once.

1, 9, 3

⑤ Check your work with a friend. There are three factors for the sentences that have 9 as the product.

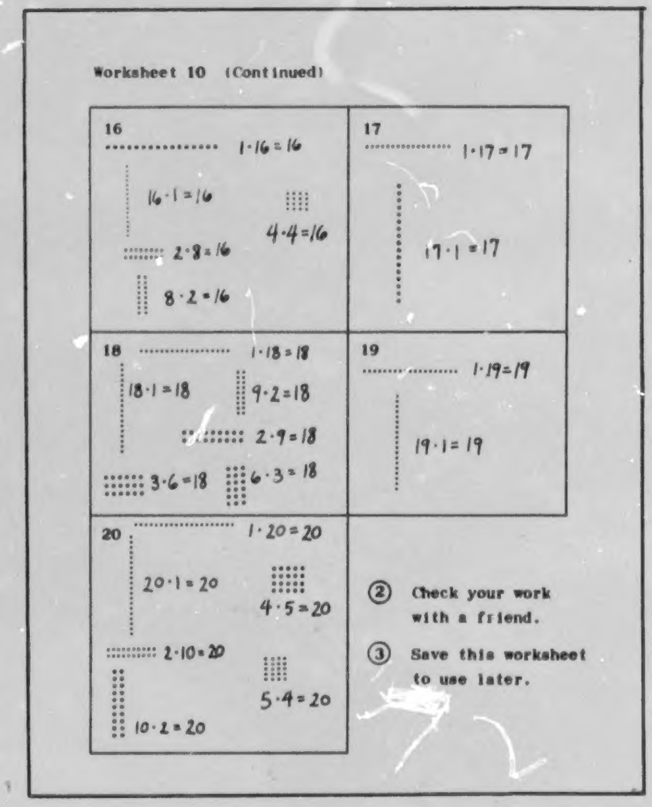

Fast Workers

Worksheet 10
Unit 27

Name ________________

① Draw as many rectangular arrays as you can for each product. Then write a multiplication sentence for each array.

Product	Sentences
11	1·11=11 11·1=11
12	1·12=12 12·1=12 2·6=12 6·2=12 3·4=12 4·3=12
13	1·13=13 13·1=13
14	1·14=14 14·1=14 2·7=14 7·2=14
15	1·15=15 15·1=15 3·5=15 5·3=15

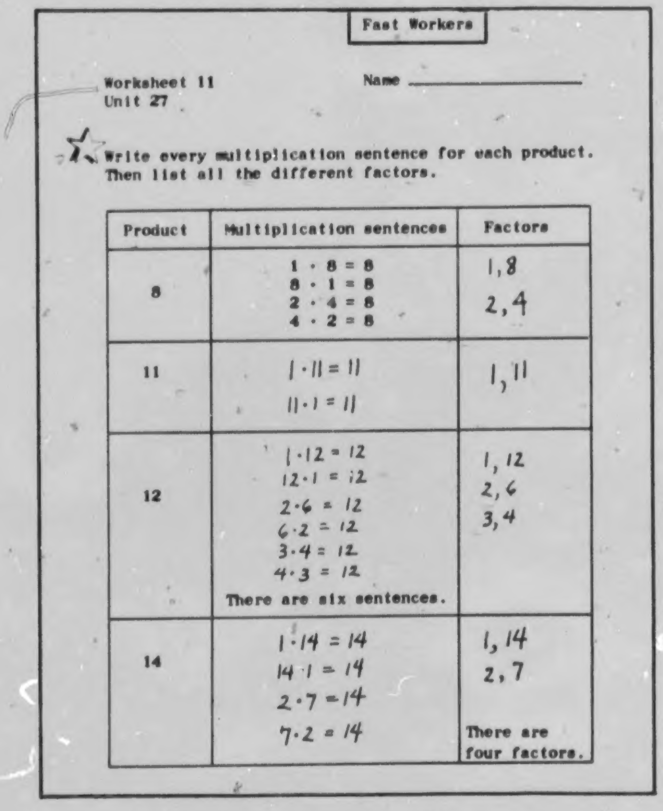

Worksheet 10 (Continued)

Product	Sentences
16	1·16=16 16·1=16 4·4=16 2·8=16 8·2=16
17	1·17=17 17·1=17
18	1·18=18 18·1=18 9·2=18 2·9=18 3·6=18 6·3=18
19	1·19=19 19·1=19
20	1·20=20 20·1=20 4·5=20 2·10=20 5·4=20 10·2=20

② Check your work with a friend.

③ Save this worksheet to use later.

Worksheets 12, 13 and 14 will be used for class discussion in the next activity. But, if some children complete the other worksheets rapidly and correctly, let them try Worksheets 12 and 13, if you wish.


Fast Workers

Worksheet 11
Unit 27

Name ________________

☆ Write every multiplication sentence for each product. Then list all the different factors.

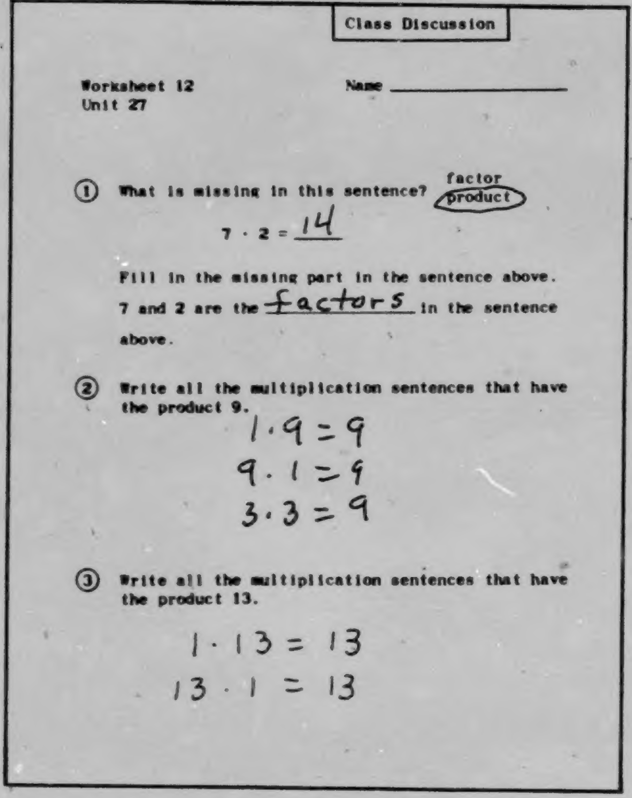
Product	Multiplication sentences	Factors
8	1 · 8 = 8 8 · 1 = 8 2 · 4 = 8 4 · 2 = 8	1, 8 2, 4
11	1 · 11 = 11 11 · 1 = 11	1, 11
12	1 · 12 = 12 12 · 1 = 12 2 · 6 = 12 6 · 2 = 12 3 · 4 = 12 4 · 3 = 12 There are six sentences.	1, 12 2, 6 3, 4
14	1 · 14 = 14 14 · 1 = 14 2 · 7 = 14 7 · 2 = 14	1, 14 2, 7 There are four factors.

Activity B

Give the children a few minutes to do Worksheets 12 and 13. After about 10 minutes, go through Worksheets 12, 13 and 14 as a class. All children should be included in this discussion.

Worksheets 12 and 13 review concepts presented on the first nine worksheets. By this time you will know any problems the children encountered in their independent work. As you go through the worksheets with the children, spend time reviewing the trouble areas so that all children will understand the work they have been doing.

On one part of Worksheet 13 the children write the different factors for each counting number (1-20). If any children have trouble doing this, suggest that they look at Worksheets 5 and 10. (Not all children will have done Worksheet 10.) Lead them to see how drawing arrays and writing multiplication sentences can help them find all the different factors for a given product.



Class Discussion

Worksheet 12
Unit 27

Name ____________________

① What is missing in this sentence? factor [✓ product]

7 · 2 = 14

Fill in the missing part in the sentence above.

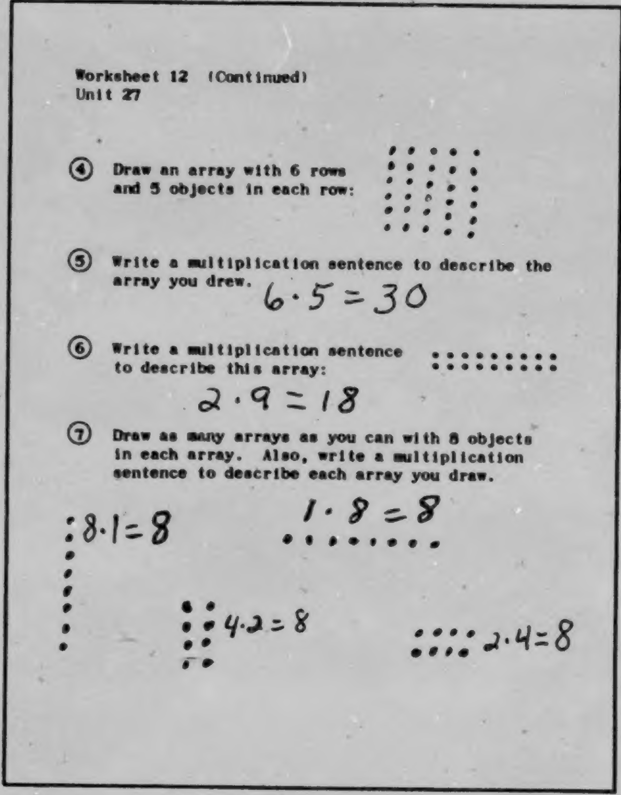
7 and 2 are the factors in the sentence above.

② Write all the multiplication sentences that have the product 9.

1·9 = 9
9·1 = 9
3·3 = 9

③ Write all the multiplication sentences that have the product 13.

1 · 13 = 13
13 · 1 = 13



Worksheet 12 (Continued)
Unit 27

④ Draw an array with 6 rows and 5 objects in each row:

⑤ Write a multiplication sentence to describe the array you drew.

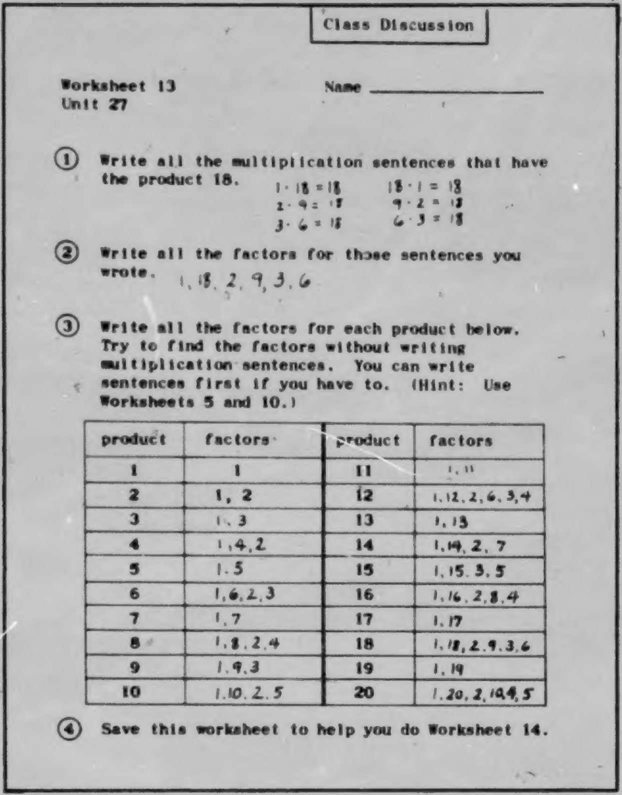
6·5 = 30

⑥ Write a multiplication sentence to describe this array:

2·9 = 18

⑦ Draw as many arrays as you can with 8 objects in each array. Also, write a multiplication sentence to describe each array you draw.

8·1=8
1·8=8
4·2=8
2·4=8



Class Discussion

Worksheet 13
Unit 27

Name ____________

① Write all the multiplication sentences that have the product 18.

1 · 18 = 18   18 · 1 = 18
2 · 9 = 18   9 · 2 = 18
3 · 6 = 18   6 · 3 = 18

② Write all the factors for those sentences you wrote. 1, 18, 2, 9, 3, 6

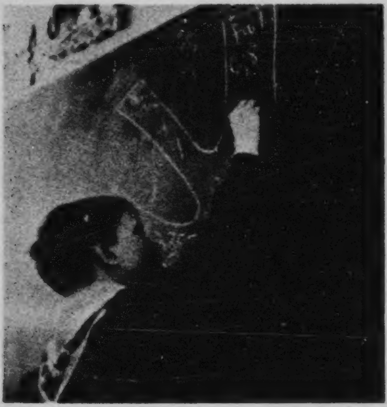
③ Write all the factors for each product below. Try to find the factors without writing multiplication sentences. You can write sentences first if you have to. (Hint: Use Worksheets 5 and 10.)

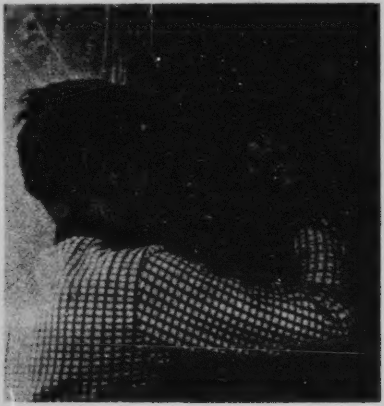
product	factors	product	factors
1	1	11	1, 11
2	1, 2	12	1, 12, 2, 6, 3, 4
3	1, 3	13	1, 13
4	1, 4, 2	14	1, 14, 2, 7
5	1. 5	15	1, 15. 3, 5
6	1, 6, 2, 3	16	1, 16, 2, 8, 4
7	1, 7	17	1, 17
8	1, 8, 2, 4	18	1, 18, 2, 9, 3, 6
9	1, 9, 3	19	1, 19
10	1, 10, 2, 5	20	1, 20, 2, 10, 4, 5

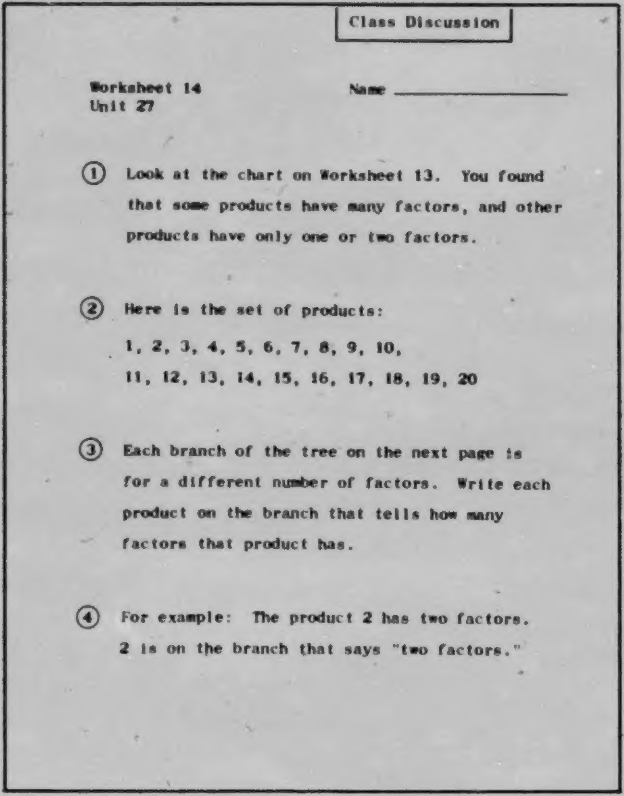
④ Save this worksheet to help you do Worksheet 14.

If some concept has been particularly difficult for the children, take time to present other examples. Activity C provides an explanation of some of the concepts. You may want to read the commentary before doing Worksheet 14. After completing Worksheets 12 and 13, you should have a good idea of how the children are doing and what areas need extra work.

On Worksheet 14 the children classify the set of counting numbers (1-20) into subsets according to the number of factors each has. The worksheet tells the children to study the chart of factors they made on the previous worksheet. If you feel the children need additional practice in writing multiplication sentences and in identifying the different factors, have the children construct another chart on the chalkboard. For each product a different child can write the multiplication sentences and list the factors. Then each child puts each product on the correct branch of his worksheet. You may want to draw the tree on the chalkboard also.







Class Discussion

Worksheet 14
Unit 27

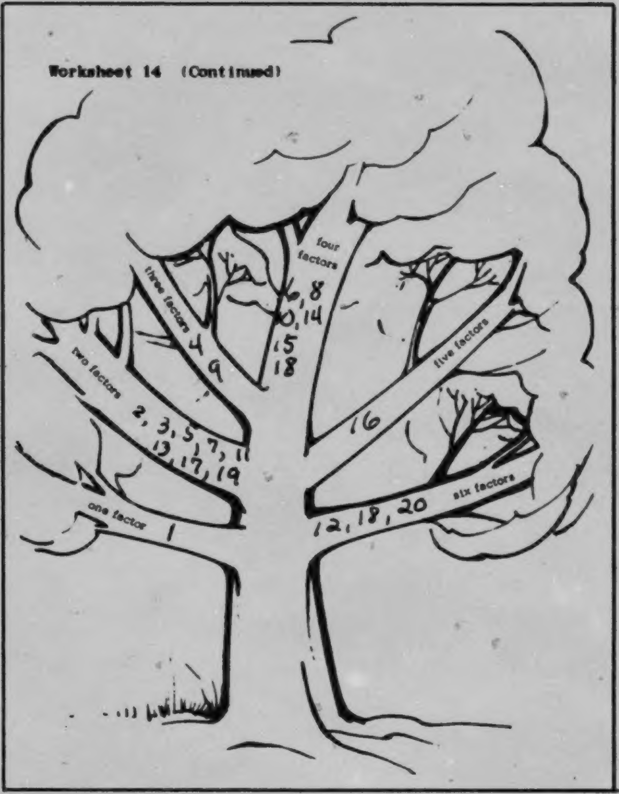
Name ________________

① Look at the chart on Worksheet 13. You found that some products have many factors, and other products have only one or two factors.

② Here is the set of products:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
11, 12, 13, 14, 15, 16, 17, 18, 19, 20

③ Each branch of the tree on the next page is for a different number of factors. Write each product on the branch that tells how many factors that product has.

④ For example: The product 2 has two factors. 2 is on the branch that says "two factors."



Worksheet 14 (Continued)

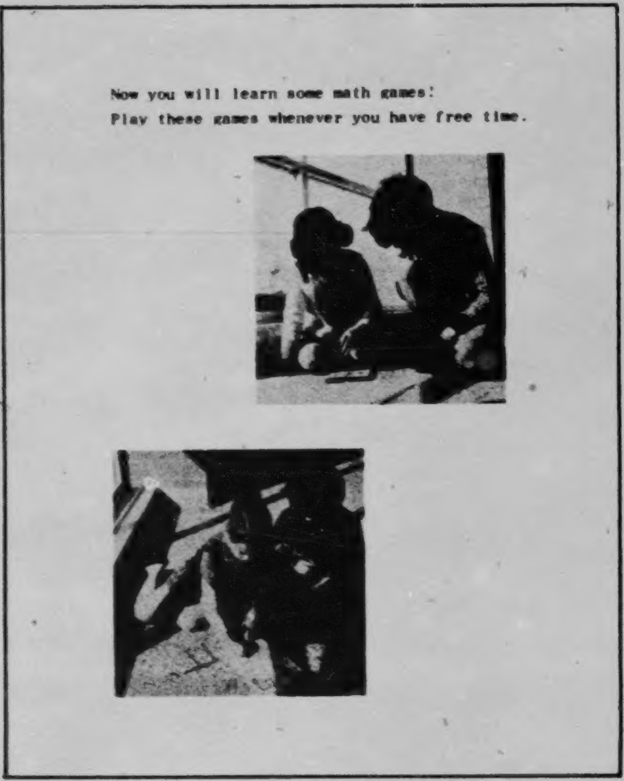
Activity C (Optional Commentary)

This commentary provides further explanation of some of the concepts presented on Worksheets 1 through 9. If the children had trouble on this first set of worksheets, these comments might be helpful for you to plan additional work for the children. If several children need additional help, you could take those children aside and go through Worksheets 10 and 11 with them. These worksheets were for fast workers, but can also provide additional practice for other children.

On Worksheets 1 and 2 the words "factor" and "product" were introduced in order to provide a convenient language for later work. The word product is used somewhat incompletely here because in the sentence $3 \cdot 4 = 12$, the expression $3 \cdot 4$ could also be called the product since it is another name for 12. The children are not given a definition of product, but are only expected to identify a product in a multiplication sentence and fill in missing products in other sentences.

The use of arrays on Worksheets 3 and 4 provided a graphic picture of the multiplication operation with whole numbers. The commutative property ($2 \cdot 3 = 3 \cdot 2$) and the multiplication property of one ($1 \cdot n = n$) should become obvious when manipulating arrays.

On Worksheet 5 the children found all the possible arrays for each number. The activity was purposely open ended so that the children had the chance to explore in a nonstructured situation. Although the student might not have found all arrays (or factors), he was encouraged to find all that he could. His previous knowledge of the multiplication facts should have proved helpful. He discovered whether or not he found all the arrays (or factors) when the class discussed Worksheets 13 and 14. Drawing arrays on previous worksheets provided a means of finding all the factors for a given product. Worksheets 6, 7, 8 and 9 gave the children practice in writing multiplication sentences for products and in isolating the factors for given products. Any children who had trouble finding all factors for a product should be encouraged to draw all the possible arrays first.



Now you will learn some math games!
Play these games whenever you have free time.

This is the instruction page from the Student Manual for Lesson 3.

Lesson 3: MATH GAMES: MULTIPLICATION, ADDITION SUBTRACTION AND COMPUTATIONAL SKILLS

The games in this lesson provide practice with basic multiplication, addition and subtraction facts, and if extended, practice with computational skills. You should devote a leisurely class period to teaching the first version of both games. The games provide a pleasant break in class routine and can be used again later in the unit whenever you feel the children need a change from the independent work they will be doing on the worksheets. For children who complete the worksheets in each lesson rapidly, you may want to suggest that they play these games in their spare time. Encourage the children to play them during free periods or whenever they have time.

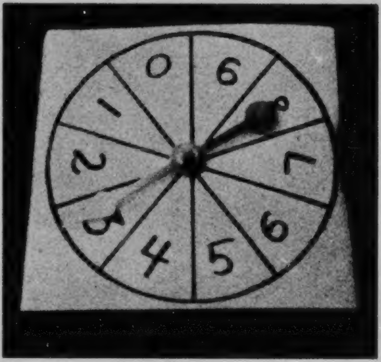
MATERIALS

-- for spinner game --

- spinners
- spinner discs, numbered 0-9
- dice, numbered 0-5 and 4-9
- paper and pencils

-- for factor game --

- paper and pencils



Assembled Spinner

PREPARATION

You or the children should get the metal spinner arrows, spinner bases and the white squares from the third-grade kit. Draw a circle on each square and divide it into ten parts. Number the parts 0 through 9. This is the disc. Assemble each spinner by putting the disc on the base and the metal arrow through the disc and into the base.

PROCEDURE

There are two games with several versions each. After the children learn the first version of each, teach them the others and encourage them to make up new versions. Do not put too much pressure on these games, but rather encourage the children to have fun while learning. You should expect a reasonable amount of noise while they are playing. If some children are playing games while others are completing the worksheets in later lessons, have the players go off to one corner of the room and remind them to play as quietly as possible so that they do not disturb the other students.

SPINNER GAME

The four versions of the spinner game provide practice with multiplication, subtraction and addition.

Version 1

Two children play this version. The players have paper, pencils and one spinner with a disc numbered 0 through 9. The first player spins the spinner twice to generate two factors. He multiplies these factors and records the product. The second player spins twice, multiplies the factors and records his product. Then the first player spins again. The game continues until each player has generated and added three products. The player with the higher score after adding three products wins the round and gets one point. The first player to win five points wins the game.

Example:

Alan	Lynne
$7 \cdot 1 = 7$	$9 \cdot 1 = 9$
$7 \cdot 6 = 42$	$2 \cdot 7 = 14$
$8 \cdot 8 = \underline{64}$	$4 \cdot 0 = \underline{0}$
113	23

Alan wins one point for this round.

The students might wish to add another rule that any player who spins 0 · 0 wins the point, and a new round is started.

The game could also be played with a pair of dice, one numbered 0 - 5 and the other 4 - 9. (Regular dice could be marked with numbers or used as they are.)

Version 2

Two or more children play, using paper, pencils and the spinner or dice. In this version the children set a goal, for example, 125. One child spins to generate two factors, multiplies them and records the product. The next child does the same. Each player adds his products after each turn and keeps a running total. The winner of the game is the player whose total gets as close as possible to the goal without going over it. If a player goes over the goal, the other player wins that game. (If several children are playing, the child who gets closest without going over wins. Any child who goes over the goal is out of the game.) If a child gets close to the goal, he may choose to "freeze" at that score and he may not spin again for the rest of that game.

For further variation, the children might wish to play the game several times. The child who comes closest to or reaches the goal each time gets one point. The first child with five points wins.

Example: Goal = 125

Bill

5 x 7 = 35
1 x 5 = 5
40
3 x 7 = 21
61
1 x 6 = 6
67
4 x 8 = 32
99
2 x 9 = 18
117
4 x 6 = 24
141

Joe

3 x 7 = 21
5 x 9 = 45
66
2 x 8 = 16
82
4 x 9 = 36
118 Freeze

Joe wins the game (or one point if they are playing several times).

Bill lost because he went over the goal.

Version 3

Two or more children play. The players choose a starting number, say, 125. This version is similar to Version 2 except that this time the children subtract the products and work back to zero. Whoever comes closest to zero (or hits it exactly) is the winner of the game. Again, the children might want to play several times, giving one point to the winner each time.

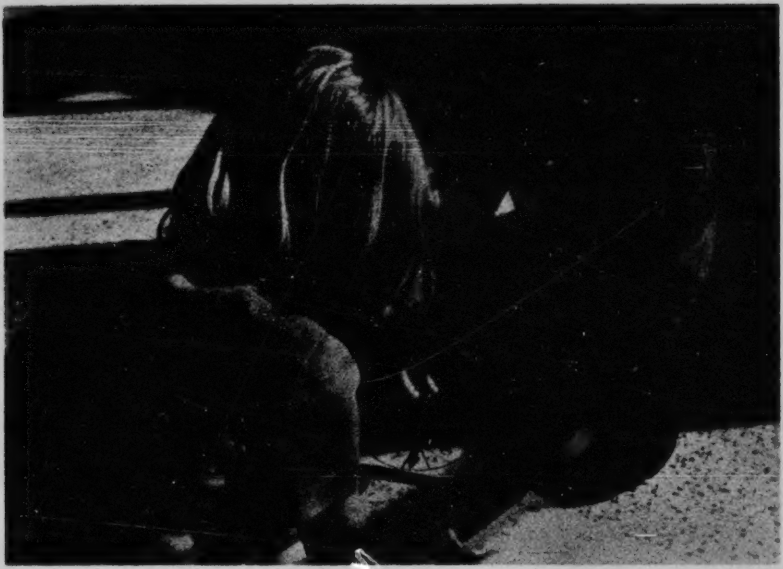
Example:

Pat	Lori
125	125
5 x 6 = -30	5 x 9 = -45
95	80
4 x 5 = -20	6 x 6 = -36
75	44
1 x 4 = - 4	3 x 8 = -24
71	20
3 x 4 = -12	4 x 5 = -20
59	0
3 x 8 = -24	
35	Lori can't be beaten — only tied!
3 x 6 = -18	
17	
4 x 4 = -16	
1	
5 x 7 = -35	

Pat went past 0. Lori wins.

Version 4

This version, called "tug-o-war," is played by two students with one sheet of paper, pencils, a spinner and a disc numbered 0 through 9. Each player, in turn, spins the spinner to generate a product. One player's goal is to reach (or pass) 100 by adding each of his products to the running total. The other player's goal is to reach (or pass) 0 by subtracting each of his products from the same total. They start with a score of 50. The players must decide who will add and who will subtract.



In the sample game below, Kim spun 7 and 1 on her first turn, so she subtracted the product, 7, from the starting score, 50. Jeanie spun 9 and 2, and added her product, 18, to 43, bringing the running total up to 61. Then Kim spun 3 and 4, and subtracted 12, which made the total 49. Then Jeanie spun 7 and 8, and added 56 to the running total. She passed her goal, 100, and won that game.

Kim (goal = 0)	Running total	Jeanie (goal = 100)
	50	
7 · 1 = 7	− 7	
	43	
	+ 18	9 · 2 = 18
	61	
3 · 4 = 12	− 12	
	49	
	+ 56	7 · 8 = 56
	105	

Jeanie passed her goal and won.

FACTOR GAME

The two versions of this game provide practice in multiplying and in writing multiplication sentences.

Version 1

Two students play this version with paper and pencils. Player A writes the whole numbers 1 through 20 on a piece of paper. Player B circles one number and then writes as many multiplication sentences as he can, with that number as the product. Player B gets one point for each sentence he wrote.

Example:

1 2 3 4 5 ⑥ 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Player B chose 6 and wrote these examples: $1 \cdot 6 = 6$, $6 \cdot 1 = 6$, $2 \cdot 3 = 6$, and $3 \cdot 2 = 6$. Player B got four points for his first turn. Now Player A chooses a number, draws a box around that number, writes the sentences and gets one point for each sentence.

Each player goes through the procedure again. After each player has had two turns, the children add their points. The player with the highest score wins that round.

Then the players play another round, each having two turns to choose a number and write the sentences. This round starts using only those numbers that had not been chosen in the first round. After two turns each, they add their points, and determine the winner of the second round. Using these 20 numbers, the children can play five rounds. The player who wins three or more rounds, wins the game.

Example:

1 2 3 4 5 ⑥ 7 8 ⑨ $\boxed{10}$ 11 $\boxed{12}$ 13 14 15 16 17 18 19 20

Bill — Kristi

$$\text{4 points}\left\{\begin{array}{l}1\cdot 6=6\\ 6\cdot 1=6\\ 2\cdot 3=6\\ 3\cdot 2=6\end{array}\right.\qquad\qquad \left.\begin{array}{r}2\cdot 5=10\\ 5\cdot 2=10\\ 1\cdot 10=10\\ 10\cdot 1=10\end{array}\right\}\text{4 points}$$

$$\text{3 points}\left\{\begin{array}{l}1\cdot 9=9\\ 9\cdot 1=9\\ 3\cdot 3=9\end{array}\right.\qquad\qquad \left.\begin{array}{r}1\cdot 12=12\\ 12\cdot 1=12\\ 2\cdot 6=12\\ 6\cdot 2=12\\ 3\cdot 4=12\\ 4\cdot 3=12\end{array}\right\}\text{6 points}$$

7 points = Bill's total

Kristi's total = 10 points

Kristi wins the first round.

Version 2

The game can be extended by using numbers larger than 20, for example, the numbers between 20 and 40. Encourage the children to think of other variations.

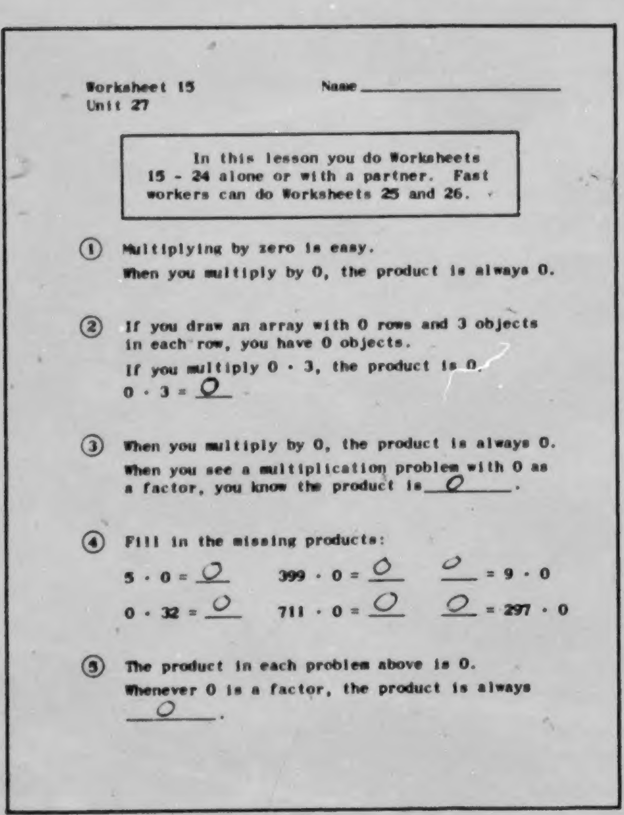
Lesson 4: MULTIPLYING BY 0 AND 1

In this lesson the children learn to multiply by 0 and 1. Worksheets and multiplication charts provide additional practice with factors and products and with simple multiplication facts. The worksheets review ordered pairs and using numbers in an ordered pair as factors in a multiplication sentence.

MATERIALS

- Worksheets 15 - 24 (independent)
- Worksheets 25 - 26 (fast workers)

PROCEDURE



Worksheet 15
Unit 27

Name ____________

In this lesson you do Worksheets 15 - 24 alone or with a partner. Fast workers can do Worksheets 25 and 26.

① **Multiplying by zero is easy.**
When you multiply by 0, the product is always 0.

② **If you draw an array with 0 rows and 3 objects in each row, you have 0 objects.**
If you multiply 0 · 3, the product is 0.
0 · 3 = 0

③ **When you multiply by 0, the product is always 0.**
When you see a multiplication problem with 0 as a factor, you know the product is 0 **.**

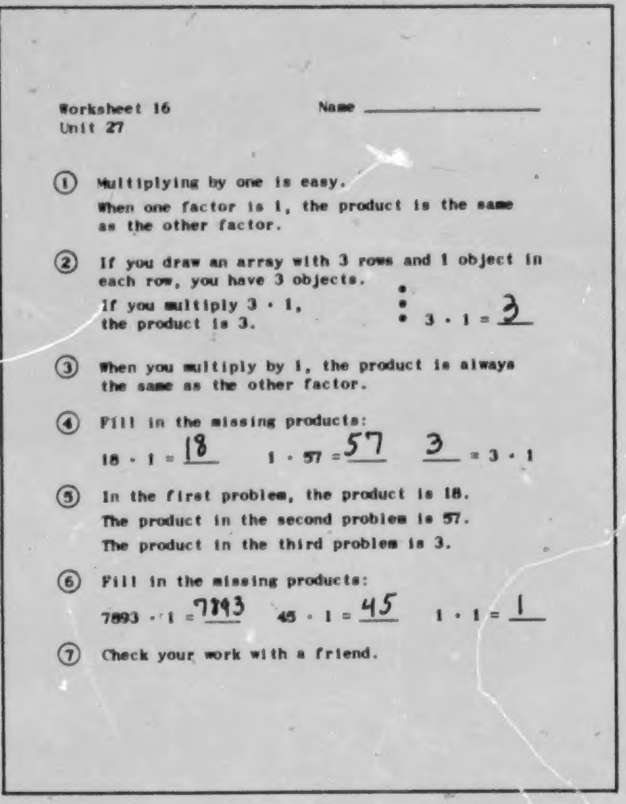
④ **Fill in the missing products:**

5 · 0 = 0 **399 · 0 =** 0 0 **= 9 · 0**
0 · 32 = 0 **711 · 0 =** 0 0 **= 297 · 0**

⑤ **The product in each problem above is 0.**
Whenever 0 is a factor, the product is always 0 **.**

The children complete Worksheets 15 through 24 independently or with partners. Fast workers should go on to Worksheets 25 and 26.

On Worksheet 15, the children learn a property of multiplying by 0. When 0 is a factor in a multiplication sentence, the product is always 0. Zero times any number is zero. The children do several problems with 0 as a factor.



Worksheet 16
Unit 27

Name ____________

① Multiplying by one is easy.
When one factor is 1, the product is the same as the other factor.

② If you draw an array with 3 rows and 1 object in each row, you have 3 objects.
If you multiply 3 · 1, the product is 3.

3 · 1 = 3

③ When you multiply by 1, the product is always the same as the other factor.

④ Fill in the missing products:

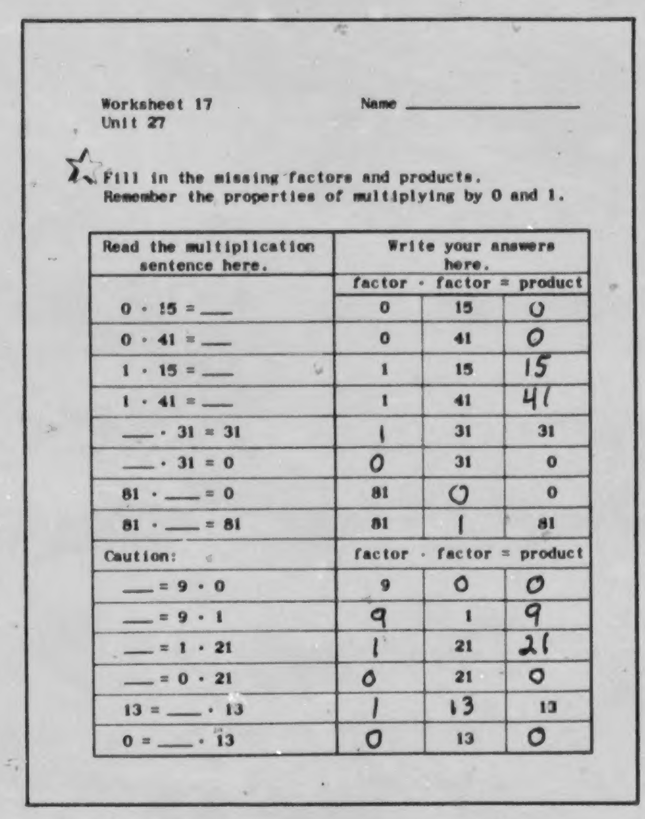
18 · 1 = 18   1 · 57 = 57   3 = 3 · 1

⑤ In the first problem, the product is 18.
The product in the second problem is 57.
The product in the third problem is 3.

⑥ Fill in the missing products:

7893 · 1 = 7893   45 · 1 = 45   1 · 1 = 1

⑦ Check your work with a friend.



Worksheet 17
Unit 27

Name ____________

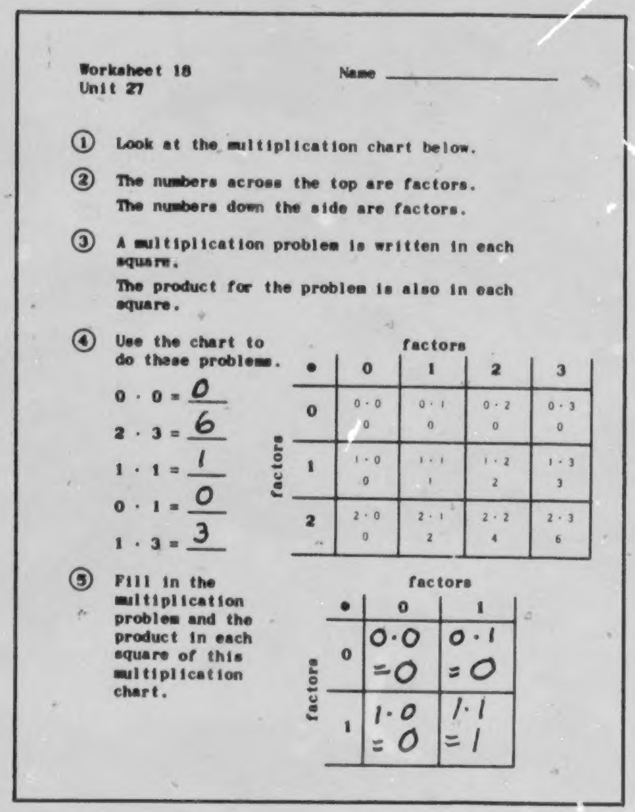
Fill in the missing factors and products.
Remember the properties of multiplying by 0 and 1.

Read the multiplication sentence here.	Write your answers here.		
	factor ·	factor =	product
0 · 15 = ___	0	15	0
0 · 41 = ___	0	41	0
1 · 15 = ___	1	15	15
1 · 41 = ___	1	41	41
___ · 31 = 31	1	31	31
___ · 31 = 0	0	31	0
81 · ___ = 0	81	0	0
81 · ___ = 81	81	1	81
Caution:	factor ·	factor =	product
___ = 9 · 0	9	0	0
___ = 9 · 1	9	1	9
___ = 1 · 21	1	21	21
___ = 0 · 21	0	21	0
13 = ___ · 13	1	13	13
0 = ___ · 13	0	13	0

Worksheet 16 studies 1 as a factor. The children discover that whenever 1 is a factor in a multiplication sentence, the product is always the same as the other factor in the sentence. The effect of multiplying by 1 is also illustrated by repeated addition and arrays. The children fill in missing products of sentences that contain 1 as a factor.

Worksheet 17 gives additional practice in multiplying by 0 and 1. Point out to the children that whenever they encounter a multiplication problem with 0 or 1 as a factor, they can quickly fill in the product using the two properties they have learned. Emphasize that the children must be alert to notice 0 and 1 as factors in problems, but that they do not need to spend much time memorizing or practicing multiplication by 0 or 1.

Worksheet 18 introduces the use of multiplication charts. The numbers across the top and along the side of the chart are factors. The numbers in the squares of the chart are products. The children study a chart with numbers 0 through 3 across the top and 0 through 2 along the side, and write the products and multiplication sentences in the squares of the chart. The children use the chart to work a few multiplication problems. Then the children fill in the multiplication sentences and products on a small chart with 0 and 1 as factors.



Worksheet 18
Unit 27

Name ____________

① Look at the multiplication chart below.

② The numbers across the top are factors.
The numbers down the side are factors.

③ A multiplication problem is written in each square.
The product for the problem is also in each square.

④ Use the chart to do these problems.

0 · 0 = 0
2 · 3 = 6
1 · 1 = 1
0 · 1 = 0
1 · 3 = 3

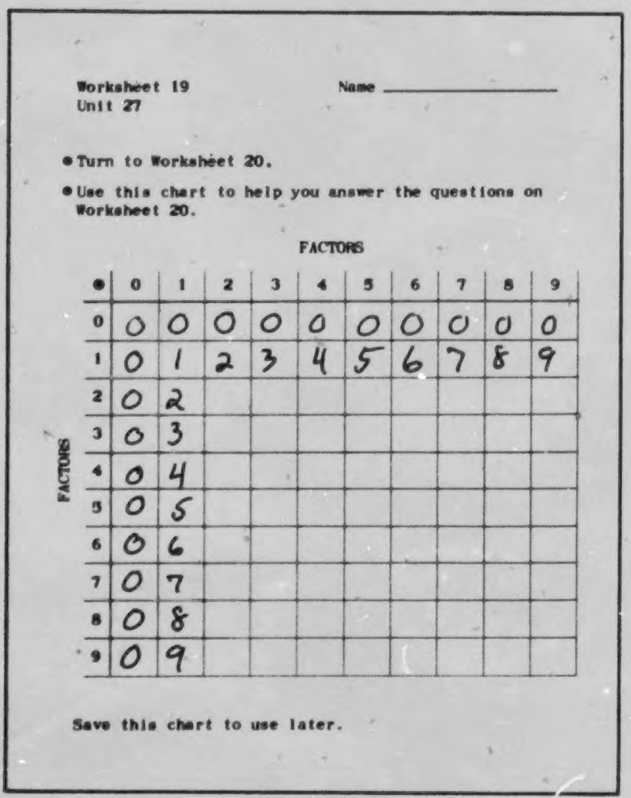
factors

•	0	1	2	3
0	0 · 0 0	0 · 1 0	0 · 2 0	0 · 3 0
1	1 · 0 0	1 · 1 1	1 · 2 2	1 · 3 3
2	2 · 0 0	2 · 1 2	2 · 2 4	2 · 3 6

⑤ Fill in the multiplication problem and the product in each square of this multiplication chart.

factors

•	0	1
0	0·0 =0	0·1 =0
1	1·0 =0	1·1 =1



Worksheet 19
Unit 27

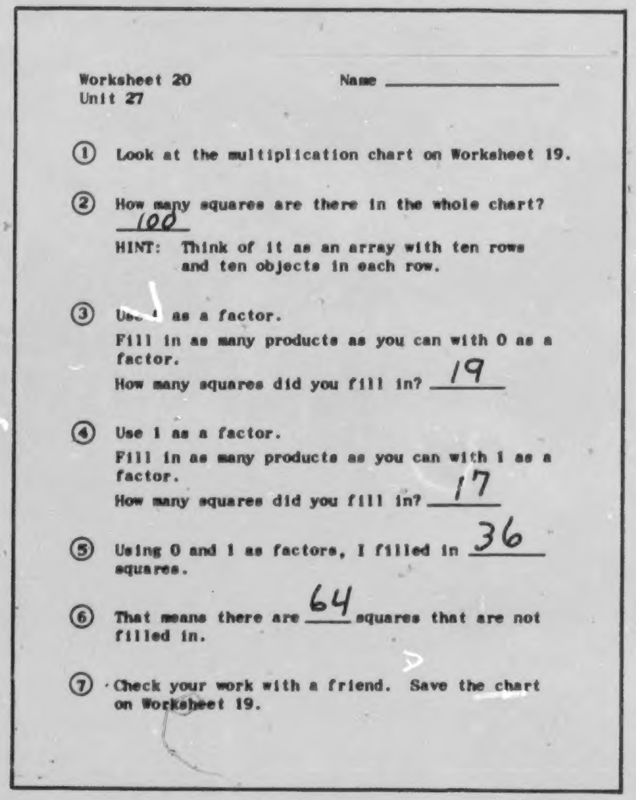
Name ____________

- Turn to Worksheet 20.
- Use this chart to help you answer the questions on Worksheet 20.

FACTORS

•	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2								
3	0	3								
4	0	4								
5	0	5								
6	0	6								
7	0	7								
8	0	8								
9	0	9								

Save this chart to use later.



Worksheet 20
Unit 27

Name ____________

① Look at the multiplication chart on Worksheet 19.

② How many squares are there in the whole chart? 100
HINT: Think of it as an array with ten rows and ten objects in each row.

③ Use 0 as a factor.
Fill in as many products as you can with 0 as a factor.
How many squares did you fill in? 19

④ Use 1 as a factor.
Fill in as many products as you can with 1 as a factor.
How many squares did you fill in? 17

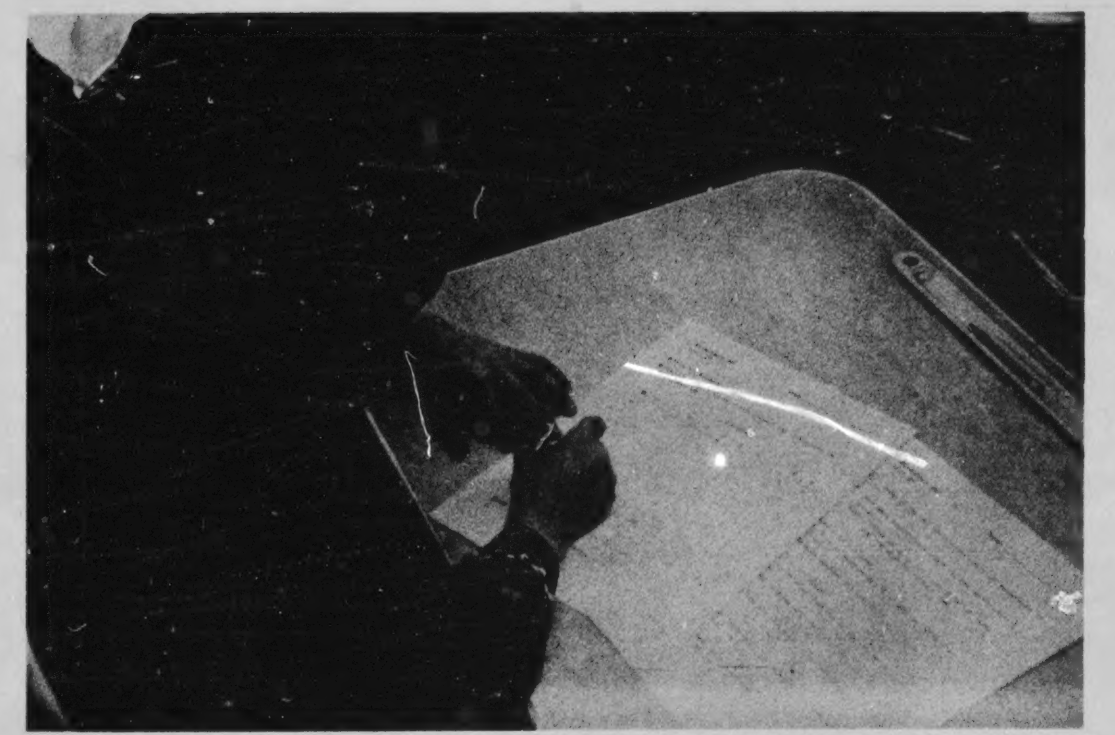
⑤ Using 0 and 1 as factors, I filled in 36 squares.

⑥ That means there are 64 squares that are not filled in.

⑦ Check your work with a friend. Save the chart on Worksheet 19.

On Worksheets 19 and 20 the children study a multiplication chart that has factors 0 through 9 along the top and side. The chart is on Worksheet 19 and some questions about the chart are on Worksheet 20. The questions on 20 tell the children to fill in squares of the chart.

First the children determine the total number of squares on the chart (100). Then they fill in all the products using 0 as a factor. These products are all zero and go in the first row and first column of the chart. (There are 19 squares with 0 as the product.) Next they fill in all products using 1 as a factor. Each product is the same as the other factor. These products go in the second row and second column of the chart. (The children fill in 17 squares using 1 as a factor.) The children see that they can fill in 36 squares of the chart using 0 and 1 as factors. Tell the children to save the chart to use later.

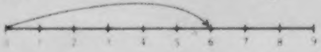


Worksheet 21 Name ____________
Unit 27

① This is an ordered pair: (1, 7).
Here is an array with 1 row and 7 objects in the row: ✱ ✱ ✱ ✱ ✱ ✱ ✱

Finish the multiplication sentence that describes the total number of objects: 1 · 7 = 7

② Here is an ordered pair: (1, 6).
Here is a number line with 1 jump and 6 spaces in a jump:

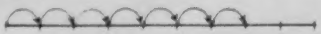


Finish the multiplication sentence that describes the total number of spaces: 1 · 6 = 6

③ Here is an array with 6 rows and 1 object in each row:

Write the ordered pair for that array: (6, 1)
Finish the multiplication sentence that describes the total number of objects in the array:
6 · 1 = 6

④ Here is a number line showing 7 jumps and 1 space in each jump:



Write the ordered pair: (7, 1)
Write the multiplication sentence that describes the total number of spaces. 7 · 1 = 7

Worksheet 22 Name ____________
Unit 27

① Here are the answers for Worksheet 21. Check your work.

box 1) 1 · 7 = 7
box 2) 1 · 6 = 6
box 3) (6, 1)
6 · 1 = 6
box 4) (7, 1)
7 · 1 = 7

② Write an ordered pair for 3 rows and 2 objects in each row. (3, 2)

Draw an array with 3 rows and 2 objects in each row.

Write a multiplication sentence to describe the array. 3 · 2 = 6

③ Write the ordered pair and the multiplication sentence for this array:

ordered pair (2, 3)
multiplication sentence 2 · 3 = 6

Worksheet 23 Name ____________
Unit 27

① (2, 4) is an ordered pair.

The first member of the pair is 2.

The second member of the pair is 4.

2 · 4 = 8

② Write a different ordered pair using 2 and 4.
(4, 2)

The first member is 4.

The second member is 2.

4 · 2 = 8

③ (3, 9) is an ordered pair. 3 · 9 = 27

Write a different ordered pair using 3 and 9. (9, 3)

Write a multiplication sentence for your ordered pair.

9 · 3 = 27

Worksheet 24 Name ____________
Unit 27

Write ordered pairs and multiplication sentences.

two numbers	ordered pairs	multiplication sentences
6 and 3	(6, 3) (3, 6)	6 · 3 = 18 3 · 6 = 18
8 and 9	(8, 9) (9, 8)	8 · 9 = 72 9 · 8 = 72
7 and 6	(7, 6) (6, 7)	7 · 6 = 42 6 · 7 = 42
4 and 6	(4, 6) (6, 4)	4 · 6 = 24 6 · 4 = 24
3 and 8	(3, 8) (8, 3)	3 · 8 = 24 8 · 3 = 24
Choose your own numbers. ____ and ____		

On Worksheets 21 and 22 the children study ordered pairs of whole numbers. Arrays and number lines illustrate graphically an interpretation of the ordered pair, and also illustrate multiplication by 1. The student applies the operation of multiplication to the ordered pair. In the first example, the children study an array of 1 row and 7 objects in the row. The ordered pair is (1,7). The numbers of the ordered pair are used as factors to produce a third number, the product: $1 \cdot 7 = 7$.

On Worksheets 23 and 24 the students again study ordered pairs, this time writing two pairs for a set of numbers. The numbers in the pair are used as factors to produce a product. The children see that two ordered pairs using the same numbers generate the same products. For example, the numbers 2 and 4 can be written in two different ordered pairs: (2,4) and (4,2). Using the numbers in each pair as factors, the children write two different multiplication sentences, but each has the same product: $2 \cdot 4 = 8$ and $4 \cdot 2 = 8$.

Worksheets 25 and 26 are for fast workers.

Worksheet 25 provides practice with simple addition, subtraction and multiplication facts. The children work each problem and then put the numbers from the problems into a puzzle that resembles a crossword puzzle.

Worksheet 26 involves multiplying by 0 and 1. With multiplication problems and word problems the children see that when the product is 0, one factor in the problem is also 0. They see that when one factor is 1 and the product is, say, 21, the other factor is 21.

Children who complete these worksheets quickly should be encouraged to play the spinner games. Or, they can fill in the remaining squares of the multiplication chart on Worksheet 19.

Fast Workers

Worksheet 25
Unit 27

Name ________________

Put one numeral in each box. Use two boxes for a 2-digit answer.

■	^{a}1	8	■	^{b}2	9	■
^{c}3	■	■	^{d}1	0	■	^{e}3
1	^{f}5	■	4	■	^{g}1	6
■	1	^{h}0	■	^{i}9	7	■
^{j}7	■	■	^{k}1	■	■	^{l}8
5	■	^{m}3	2	■	■	2
■	^{n}1	8	■	^{o}6	7	■

Across

a. 2 · 9 = ____
b. 35 - 6 = ____
c. ____ · 4 = 12
d. 5 · 2 = ____
e. 3 · 1 = ____
f. 3 · ____ = 15
g. 4 · 4 = ____
h. 368 · 0 = ____
i. 97 · 1 = ____
j. 12 - 5 = ____
l. 2 · 4 = ____
m. 51 - 19 = ____
n. 3 · 6 = ____
0. 100 - 33 = ____

Down

b. 4 · 5 = ____
c. 15 + 16 = ____
d. 2 · 7 = ____
e. 18 + 18 = ____
f. 60 - 9 = ____
g. 9 + 8 = ____
j. 49 + 26 = ____
k. 2 · 6 = ____
l. 14 + 19 + 32 + 17 = ____
m. 19 + 19 = ____

Fast Workers

Worksheet 26
Unit 27

Name ________________

0 · 89 = 0

When one factor is 0, the product is always 0.

0 · 13 = 0

When the product is 0, one factor is 0.

0 · 0 = 0

If two factors are 0, the product is 0.

1 · 8765 = 8765

When one factor is 1, the product is always the same as the other factor.

If one factor is 1 and the product is 21, what is the other factor? 21

If one factor is 1 and the product is 937, what is the other factor? 937

Lesson 5: USING A LETTER AS A PLACEHOLDER FOR A NUMBER

In this lesson the children are introduced to the idea of using a letter as a placeholder for a number. First they use letters in addition problems ($n = 6$, $1 + n = ?$) and then in multiplication problems (if $n = 2$, then $n \cdot 3 = ?$). The worksheets review multiplication by 0 and 1, using letters as factors, writing ordered pairs, drawing arrays for ordered pairs, writing multiplication sentences to describe arrays, and commutativity of multiplication.

Now you can do Worksheets 27, 28 and 29.
Fast workers can also do Worksheet 30.

Do not do any worksheets marked "Class Discussion," until your teacher tells you to.

MATERIALS

- Worksheets 27-29 (independent)
- Worksheet 30 (fast workers)
- Worksheets 31-33 (discussion)

PROCEDURE

In Activity A, Worksheets 27, 28, and 29 are completed independently or with partners. Worksheet 30 is for fast workers.

In Activity B you discuss Worksheets 31, 32 and 33 as a group.

Activity A

Worksheet 27 introduces the use of a letter as a placeholder for a number. Examples are given for addition problems. The purpose of the introduction is to familiarize the children with the use of a letter in place of a number, and to help them discover how to determine what number a letter represents, and how to determine the sum of a letter plus a number when they have been told the value of that letter. In the first example the children are told that n equals 6. If $n = 6$, then $1 + n$ is equivalent to $1 + 6$. The sum is 7. There are other similar examples. The children are given some of the answers at the bottom of the second page so that they can check their work.

Worksheet 27
Unit 27

Name ____________________

① You can use a letter in place of a number.
Let's try some addition problems using letters.

② Let's say that n = 6.
1 + n = ?
1 + 6 = 7
1 + n = 7

③ Let's say that b = 2.
b + 3 = ?
2 + 3 = 5
b + 3 = 5

④ Let's say x = 3.
x + 5 = ?
3 + 5 = 8
number
x + 5 = 8

⑤ If p = 4, then p + p = 8
Check this answer with a friend.

Worksheet 27 (Continued)

⑥ Here are more addition problems using letters in place of numbers.
Find the sums. Add.
Rewrite each problem using numbers if you have to.

⑦ If r = 3, then r + 4 = 7 (Hint: 3 + 4 = ?)
If p = 5, then p + 10 = 15
If x = 7, then x + 20 = 27
If d = 6, then 11 + d = 17
If h = 3, then 7 + h = 10

⑧ Check your work. If you got any wrong answers, read this worksheet again.
Here are the answers: 7, 15, 27, 17, 10.

Worksheet 28
Unit 27

Name ____________________

① A letter can replace a factor in a multiplication problem.
In these problems we will use letters as factors.

② Let's say that n = 6.
Here is how you find the product of 1 · n, when n = 6.

n = 6
1 · n = ?
1 · n = 1 · 6
1 · 6 = 6
1 · n = 6

③ Let's say that a = 3.
1 · a = ?
Find the product.

a = 3
1 · a = ?
1 · a = 1 · 3
(number)
1 · 3 = 3
(number)
1 · a = 3

④ Find the missing products.
Rewrite the problem using the number in place of the letter only if you have to. Try not to rewrite the problem.

If x = 4, then 1 · x = 4
If n = 2, then n · 3 = 6
If x = 7, then 2 · x = 14
If y = 4, then y · 3 = 12

Worksheet 28 (Continued)

⑤ If you had trouble finding the products to the problems in number 4, read the first page of this worksheet again.

The answers to the problems in number 4 are:

If x = 4, then 1 · x = 4
If n = 2, then n · 3 = 6
If x = 7, then 2 · x = 14
If y = 4, then y · 3 = 12

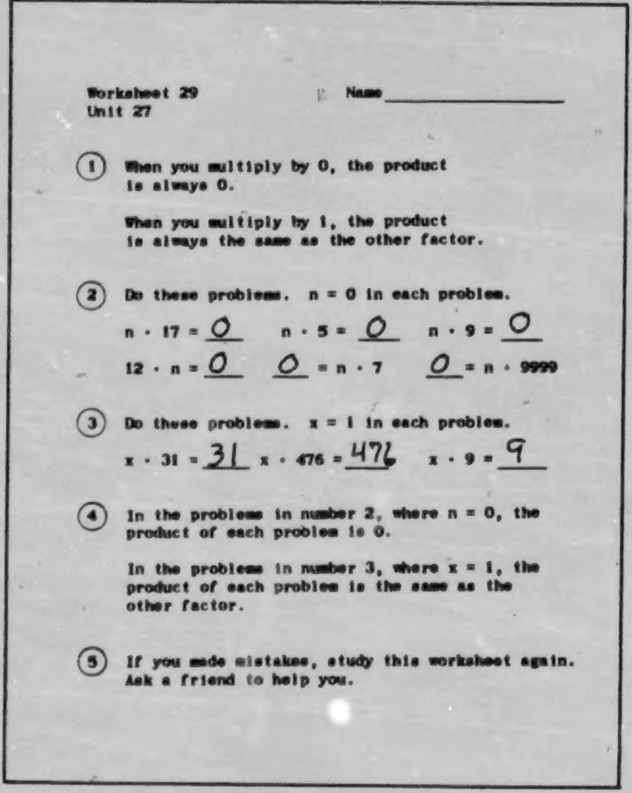
⑥ Find the missing products. Try to work the problems without rewriting them.
If z = 10, then z · 2 = 20
If m = 7, then 2 · m = 14
If d = 9, then d · 3 = 27
If q = 3, then q · q = 9
If x = 3, then x · 7 = 21
If n = 8, then 3 · n = 24

⑦ Check your work with a friend.
If you made mistakes, try the problems again.

On Worksheet 28 the children begin using letters as factors in multiplication problems. The children are told the value of a letter and then are expected to use that letter as a factor to determine the product.

Worksheet 29 reviews the properties of multiplying by 0 and 1 in the context of using a letter in place of a number. For example, the children are given several problems where $n = 0$. n is a factor in each problem and the children must find the product, which is 0 in each case. In other problems the children are told that $x = 1$. x is a factor in each problem. The product is always the same as the other factor in the problem.

Worksheet 30 is for fast workers. If the children have trouble with Worksheets 28 and 29, you may want all the children to do this worksheet. It provides additional practice in multiplying using letters as factors.



Worksheet 29
Unit 27

Name ____________

① When you multiply by 0, the product is always 0.

When you multiply by 1, the product is always the same as the other factor.

② Do these problems. $n = 0$ in each problem.

$n \cdot 17 = 0$ $n \cdot 5 = 0$ $n \cdot 9 = 0$

$12 \cdot n = 0$ $0 = n \cdot 7$ $0 = n \cdot 9999$

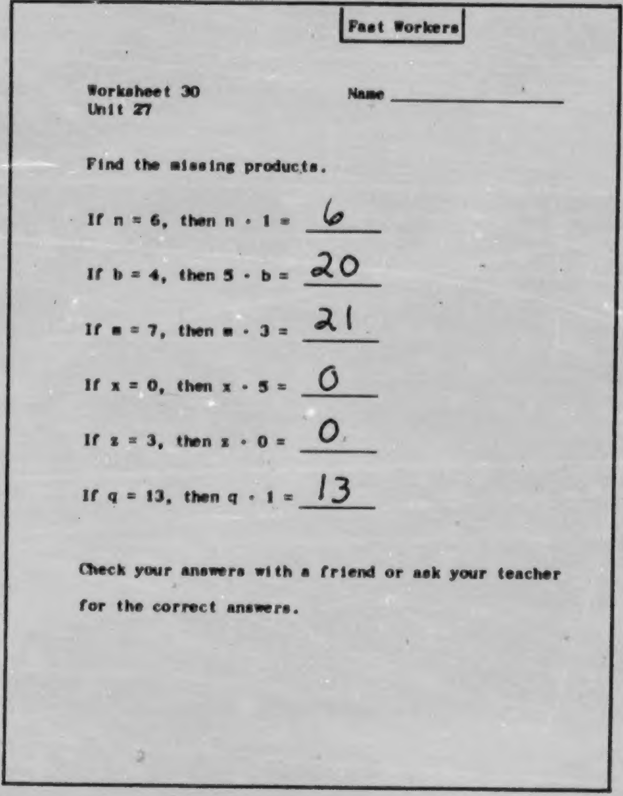
③ Do these problems. $x = 1$ in each problem.

$x \cdot 31 = 31$ $x \cdot 476 = 476$ $x \cdot 9 = 9$

④ In the problems in number 2, where $n = 0$, the product of each problem is 0.

In the problems in number 3, where $x = 1$, the product of each problem is the same as the other factor.

⑤ If you made mistakes, study this worksheet again. Ask a friend to help you.



Fast Workers

Worksheet 30
Unit 27

Name ____________

Find the missing products.

If $n = 6$, then $n \cdot 1 = 6$

If $b = 4$, then $5 \cdot b = 20$

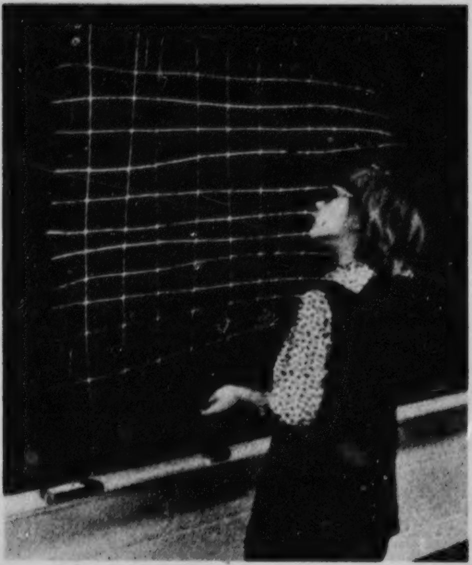
If $m = 7$, then $m \cdot 3 = 21$

If $x = 0$, then $x \cdot 5 = 0$

If $z = 3$, then $z \cdot 0 = 0$

If $q = 13$, then $q \cdot 1 = 13$

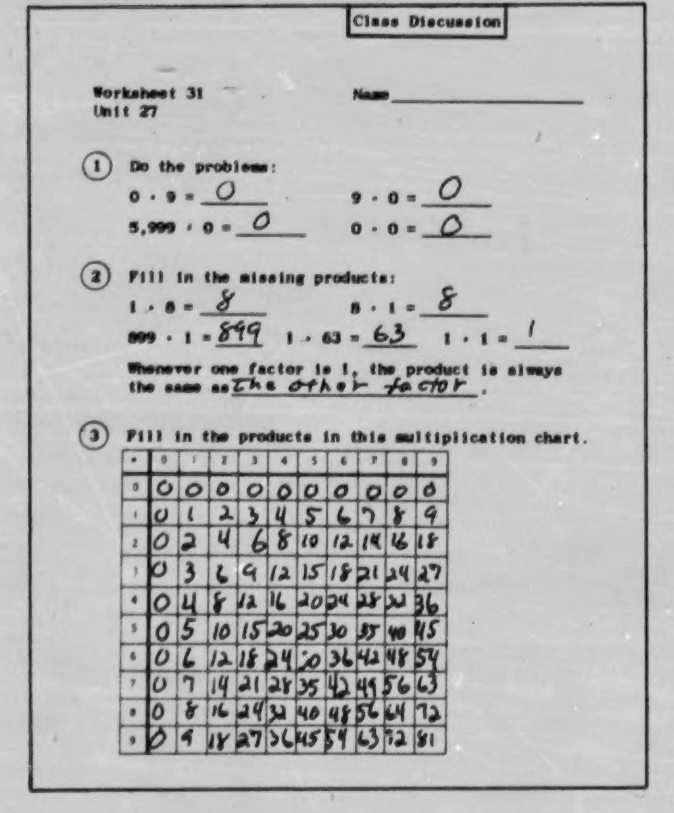
Check your answers with a friend or ask your teacher for the correct answers.



Activity B

Give the children a few minutes to do Worksheets 31, 32 and 33. Then, as a class, discuss these worksheets.

Worksheet 31 includes a review of multiplication by 0 and 1. The children work problems involving factors of 0 and 1, and they review the properties of multiplying by these factors. Ask different children to give the answers and reasons for answers to the problems. The worksheet also includes a multiplication chart that gives practice in multiplying by 0 through 9. You may want to draw this chart on the chalkboard and ask different children to come to the board to fill in the squares.



Class Discussion

Worksheet 31
Unit 27

Name ____________

① Do the problems:

$0 \cdot 9 =$ 0
$9 \cdot 0 =$ 0
$5{,}999 \cdot 0 =$ 0
$0 \cdot 0 =$ 0

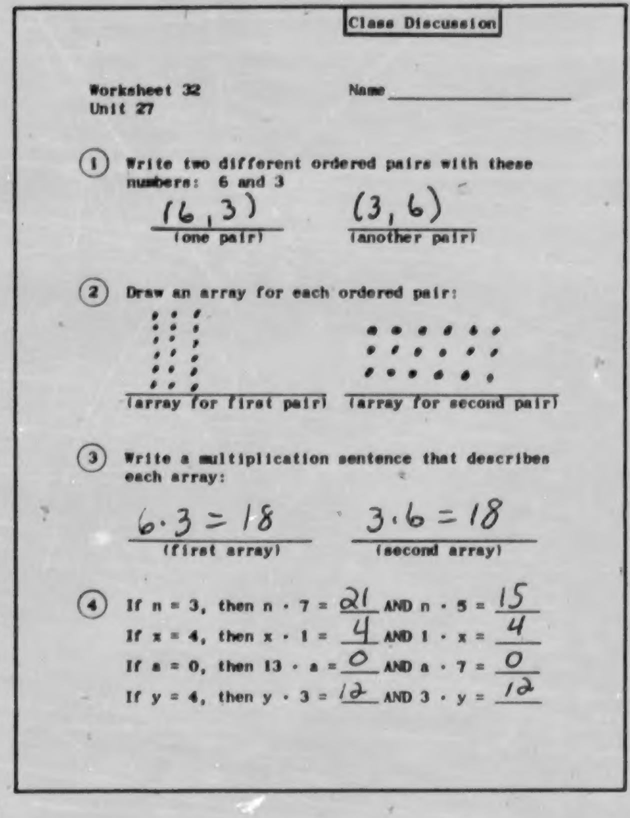
② Fill in the missing products:

$1 \cdot 8 =$ 8
$8 \cdot 1 =$ 8
$899 \cdot 1 =$ 899
$1 \cdot 63 =$ 63
$1 \cdot 1 =$ 1

Whenever one factor is 1, the product is always the same as the other factor.

③ Fill in the products in this multiplication chart.

·	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81



Class Discussion

Worksheet 32
Unit 27

Name ____________

① Write two different ordered pairs with these numbers: 6 and 3

(6, 3) (one pair)
(3, 6) (another pair)

② Draw an array for each ordered pair:

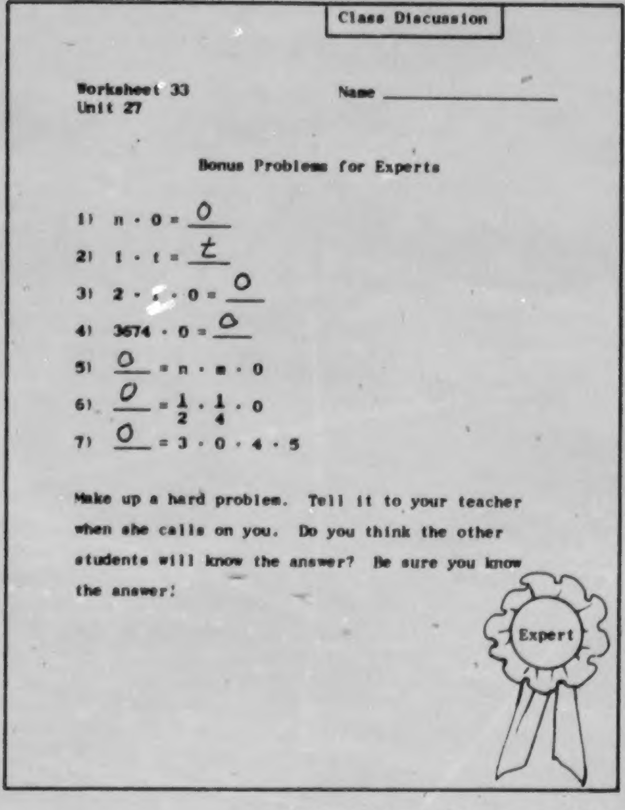
(array for first pair)
(array for second pair)

③ Write a multiplication sentence that describes each array:

$6 \cdot 3 = 18$ (first array)
$3 \cdot 6 = 18$ (second array)

④ If $n = 3$, then $n \cdot 7 =$ 21 AND $n \cdot 5 =$ 15
If $x = 4$, then $x \cdot 1 =$ 4 AND $1 \cdot x =$ 4
If $a = 0$, then $13 \cdot a =$ 0 AND $a \cdot 7 =$ 0
If $y = 4$, then $y \cdot 3 =$ 12 AND $3 \cdot y =$ 12

On Worksheet 32 you discuss with the children the concepts of writing ordered pairs, drawing arrays for those pairs, and writing multiplication sentences to describe the arrays. Also on this worksheet, the children get additional practice in using a letter in place of a number. The problems give practice in multiplying by 0 and 1 (with a letter as a number) and also provide examples of the commutativity of multiplication.



Class Discussion

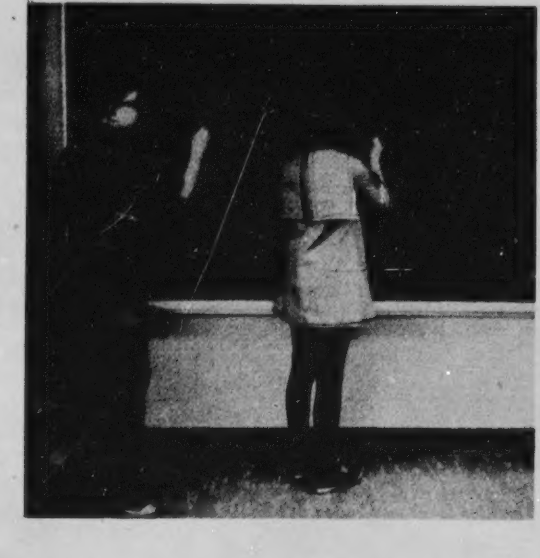
Worksheet 33
Unit 27

Name ____________

Bonus Problems for Experts

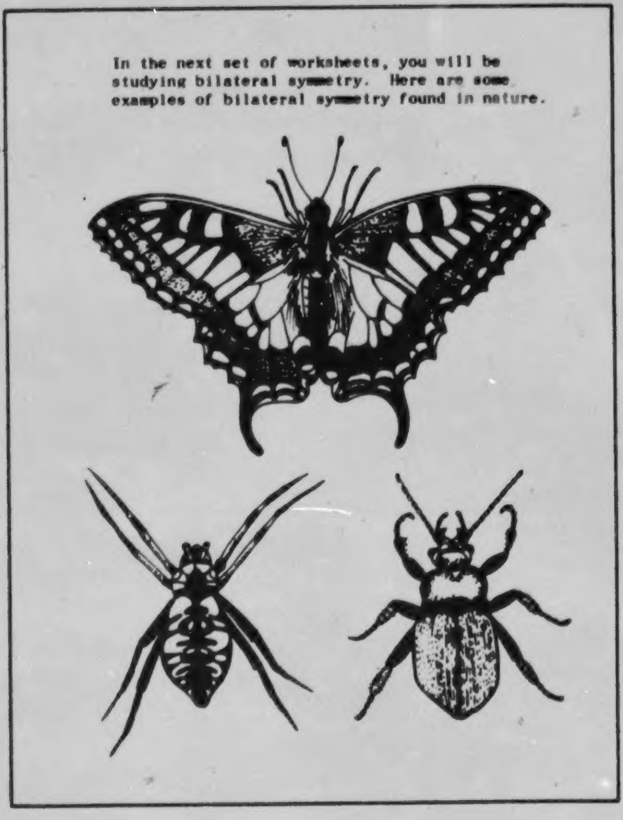
1) $n \cdot 0 = \underline{0}$
2) $1 \cdot t = \underline{t}$
3) $2 \cdot 1 \cdot 0 = \underline{0}$
4) $3674 \cdot 0 = \underline{0}$
5) $\underline{0} = n \cdot m \cdot 0$
6) $\underline{0} = \frac{1}{2} \cdot \frac{1}{4} \cdot 0$
7) $\underline{0} = 3 \cdot 0 \cdot 4 \cdot 5$

Make up a hard problem. Tell it to your teacher when she calls on you. Do you think the other students will know the answer? Be sure you know the answer!



Worksheet 33 is called "Bonus Problems for Experts." Discuss these problems as a class, but let the children give you the answers before you tell them whether or not they are correct. Encourage all the children to try to be experts! If the children have trouble with a particular problem, remind them of the property that applies to that problem. In the first problem, n · 0 = ____, the children have not been told what n equals. But they do not need to know, since whenever 0 is a factor, the product is 0. In the third problem, 2 · 1 · 0, suggest that the children multiply two factors at a time: 2 · 1 = 2; 2 · 0 = 0. The children should see that no matter how many factors are included in the problem, if one of these factors is 0, the product is 0. In the sixth problem, the children may wonder how to multiply fractions. Lead them to realize that because one factor is 0, the product is zero. They do not need to know the product of one-half times one-fourth.

At the bottom of the worksheet, the children are told to make up a problem. Call on the children to give their problems. Ask other children to tell the products.



In the next set of worksheets, you will be studying bilateral symmetry. Here are some examples of bilateral symmetry found in nature.

This is the instruction page from the Student Manual for Lesson 6.

Lesson 6: BILATERAL SYMMETRY

In this lesson the students study bilateral symmetry in pictures and in some addition and multiplication charts. The children measure angles with their clock protractors. They discover that when the factors are arranged in numerical order on a multiplication chart, the products possess bilateral symmetry.

MATERIALS

- rulers
- scissors
- clock protractors (from Unit 26)
- Worksheets 34-39 (discussion)

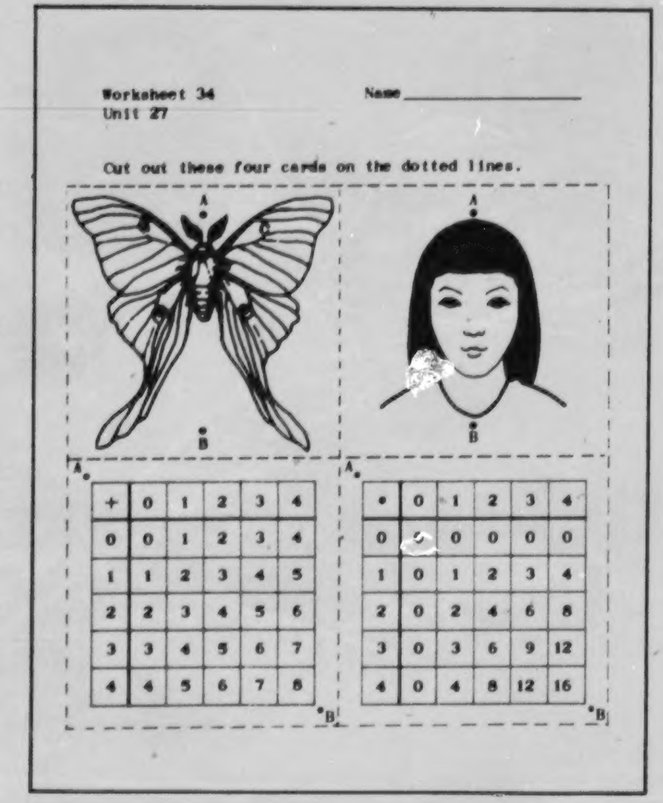
PROCEDURE

In Activity A the children complete Worksheets 34 through 39 as a group. Activity B is optional commentary concerning bilateral symmetry.

Activity A

On Worksheet 34 the children cut out two pictures, a multiplication chart, and an addition chart. As a group they follow the instructions on Worksheet 35. They will discover that each card has the common property of bilateral symmetry.

On the addition chart, squares that correspond because of symmetry also contain the same entry. For example, the square containing the entry for 2 + 1 corresponds to the square containing the entry for 1 + 2. Both of these squares contain the number 3. This observation relates the commutative property of addition (and multiplication) to symmetry of the addition (and multiplication) table. This observation is not made explicit for the students.

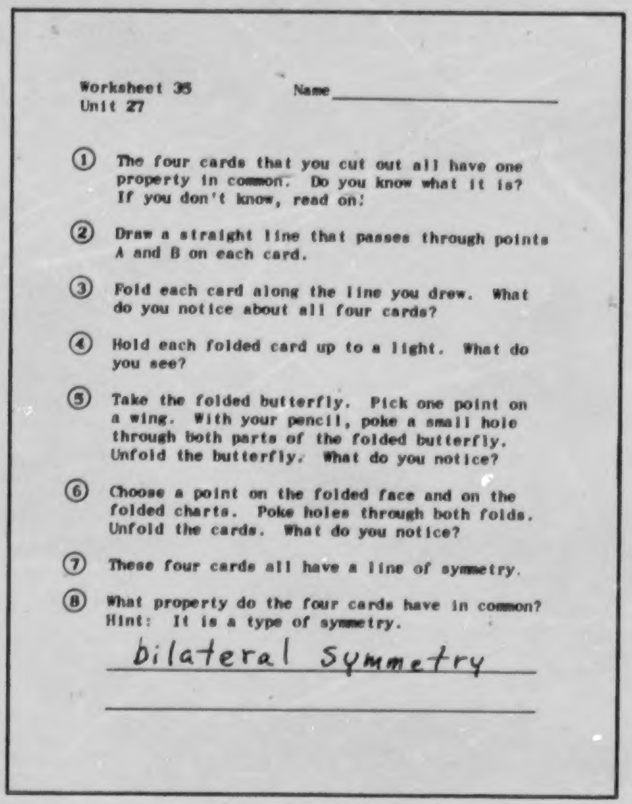


Worksheet 34 Name ____________
Unit 27

Cut out these four cards on the dotted lines.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	5
2	2	3	4	5	6
3	3	4	5	6	7
4	4	5	6	7	8

•	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	6	8
3	0	3	6	9	12
4	0	4	8	12	16



Worksheet 35 Name ____________
Unit 27

① The four cards that you cut out all have one property in common. Do you know what it is? If you don't know, read on!

② Draw a straight line that passes through points A and B on each card.

③ Fold each card along the line you drew. What do you notice about all four cards?

④ Hold each folded card up to a light. What do you see?

⑤ Take the folded butterfly. Pick one point on a wing. With your pencil, poke a small hole through both parts of the folded butterfly. Unfold the butterfly. What do you notice?

⑥ Choose a point on the folded face and on the folded charts. Poke holes through both folds. Unfold the cards. What do you notice?

⑦ These four cards all have a line of symmetry.

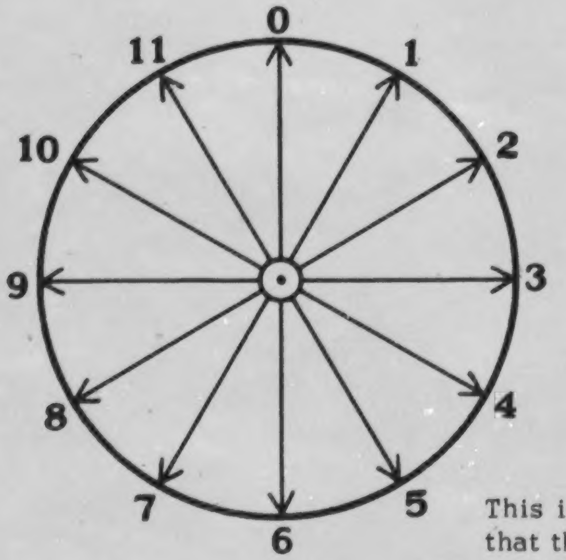
⑧ What property do the four cards have in common? Hint: It is a type of symmetry.

bilateral symmetry

Worksheet 36 provides instructions for Worksheet 37, which is a multiplication chart with the products arranged along a line of bilateral symmetry.

The children find the product entries for 3 · 7 and 7 · 3, and draw a line that passes through these product entries (and through points A and B). The students measure the four angles formed by line AB which intersects the line of bilateral symmetry (line MN). Each angle has a mag of 3 o'clock (90°). The children are asked to draw other lines that intersect line MN. Each of the angles formed by these lines also has a mag of 3 o'clock.

(Activity B includes some optional commentary about bilateral symmetry. For more information about symmetry, see Minnemast Unit 14, Exploring Symmetrical Patterns.)



This is the clock protractor that the children use on the worksheets in this lesson.

Worksheet 36 Name____________________
Unit 27

① Tear out Worksheet 37.

② 3 · 7 and 7 · 3 = 21

③ Find the squares on the chart that show the product of 3 · 7 and 7 · 3.

④ Point A is in the box with the product entry 21.
Point B is in the box with the product entry 21.

⑤ Draw a straight line that passes through points A and B. Use a ruler.

⑥ $\overleftrightarrow{MN}$ is the line of symmetry. $\overleftrightarrow{MN}$ intersects $\overleftrightarrow{AB}$. Four angles are formed.

⑦ Measure each angle with your clock protractor. The mag of each angle equals 3 o'clock.

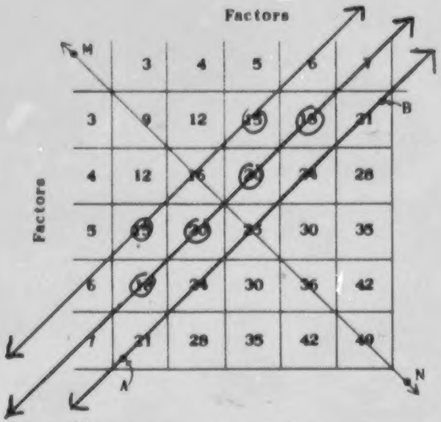
⑧ Circle the products for these pairs of factors.

6 · 3 and 3 · 6
4 · 5 and 5 · 4
3 · 5 and 5 · 3

Draw lines to connect each pair of products. Your lines should intersect $\overleftrightarrow{MN}$. Measure the angles with your clock protractor. What is the mag of each angle: 3 o'clock.

Worksheet 37 Name____________________
Unit 27

Multiplication Chart

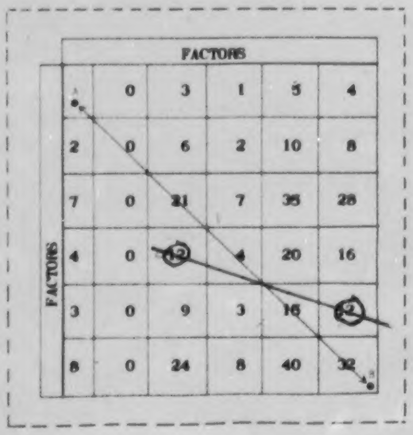


	3	4	5	6	7
3	9	12	15	18	21
4	12	16	20	24	28
5	15	20	25	30	35
6	18	24	30	36	42
7	21	28	35	42	49

$\overleftrightarrow{MN}$ is the line of symmetry.

Worksheet 38 Name____________________
Unit 27

Cut out this chart on the dotted lines.
Turn to the next worksheet.



	0	3	1	5	4
2	0	6	2	10	8
7	0	21	7	35	28
4	0	12	4	20	16
3	0	9	3	15	12
8	0	24	8	40	32

Worksheet 39 Name____________________
Unit 27

1. Does the chart on Worksheet 38 have a line of bilateral symmetry? YES [✓ NO]

2. Test your answer. Cut out the chart. Fold along line AB and test for bilateral symmetry.

3. Circle the two product entries for 4 · 3 and 3 · 4.
Draw a line segment connecting these products.
Use your clock protractor to measure the angles.

4. The multiplication chart on Worksheet 37 was bilaterally symmetric.
The chart on Worksheet 38 is not bilaterally symmetric.
What makes them different?
The factors must be arranged in the same numerical order along top and side

Worksheets 38 and 39 involve work with a multiplication chart with factors that are not in numerical order. Also, the factors along the top are not the same as those along the side of the chart. Therefore, the products of the chart do not possess bilateral symmetry. The children cut out the chart on Worksheet 38 and then follow the instructions on Worksheet 39.

Activity B (Optional Commentary)

This commentary is provided for your use in understanding bilateral symmetry (symmetry about a line or plane).

Consider the letter A in Figure 1. Any line drawn perpendicular to the dotted line in the figure cuts the pattern into pairs of points that are equidistant from the dotted line. For example, the points a and b in Figure 2 (on the next page) are at equal distances from the dotted line.

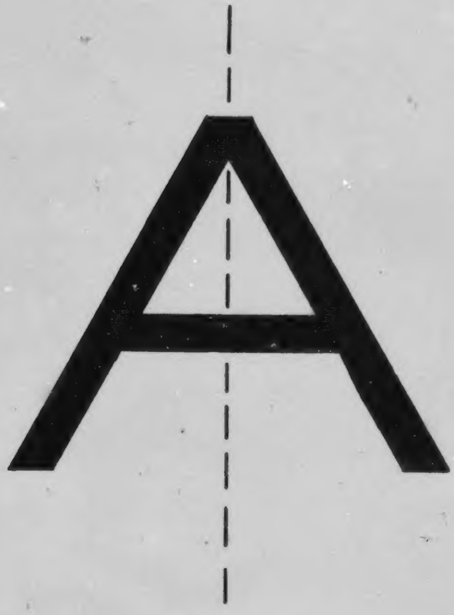


Figure 1

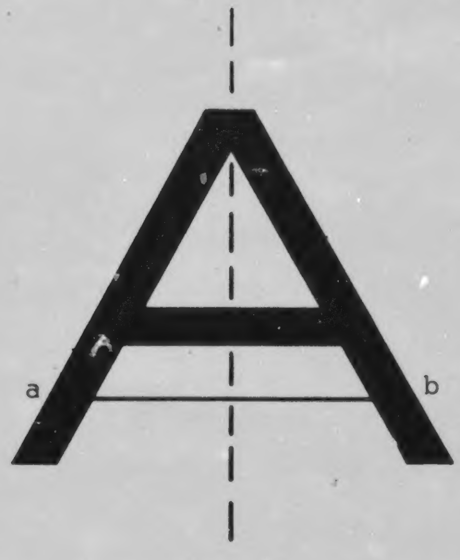


Figure 2

The letter A is said to exhibit symmetry about a line, or bilateral symmetry. The dotted line is called the line of symmetry or the axis of symmetry.

Any pattern through which a line can be drawn such that every line perpendicular to this line cuts the pattern into pairs of points equidistant from the line is said to exhibit symmetry about a line or bilateral symmetry.

A simpler but less exact statement of this idea is that for every point on one side of the line of symmetry there is a corresponding point on the other side. The paired points must be at equal distances from the line of symmetry. Figure 3 (on the last page of this lesson) shows some bilaterally symmetric patterns that occur in nature. There are tests one can use to determine whether or not a pattern possesses bilateral symmetry. Two of these tests are described below.

If a mirror is placed in a vertical position along the dotted line in Figure 1, the pattern viewed (half of A and its image in the mirror) coincides precisely with the original letter A. The mirror test consists of holding a mirror on the axis of symmetry to show that the pattern can be divided into two parts identical in size, one part being the mirror image of the other. In any bilaterally symmetric pattern, one half of the pattern is the mirror image of the other half. This is the reason that bilateral symmetry is also referred to as mirror symmetry. It is interesting to test some non-symmetric patterns with a mirror.

Bilaterally symmetric patterns may also be folded along the axis of symmetry to test for the coincidence of the two halves of the pattern. This gives another test, the folding test, for bilateral symmetry.

There are some differences in examining the symmetry of patterns on plane surfaces and of three-dimensional objects. Corresponding to the line of symmetry of plane patterns, bilaterally symmetric three-dimensional objects have a plane of symmetry. Every line perpendicular to this plane cuts the figure into pairs of corresponding points that are at equal distances from the plane. Obviously, a solid object cannot be folded to test for bilateral symmetry. However, a mirror can be placed along the plane of symmetry to show the existence of bilateral symmetry. The human face is an interesting object that has approximate symmetry with respect to a plane. Other objects that might be tested are a cone, a beetle, and toy figures of people or animals.

A picture of a three-dimensional object is, of course, a two-dimensional pattern. Thus the symmetry of a picture of a three-dimensional object may be tested by either the mirror or folding test.

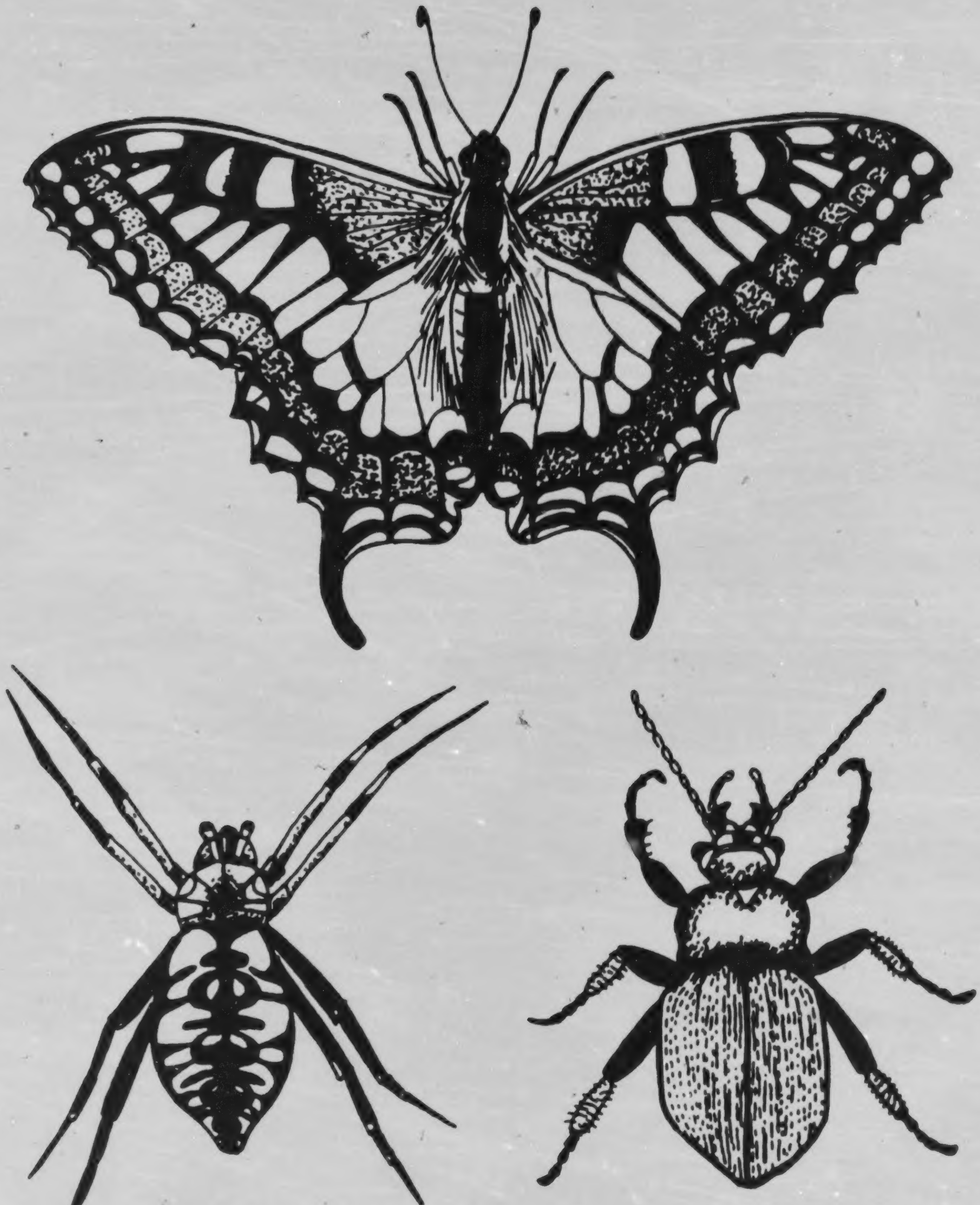


Figure 3

Lesson 7: MULTIPLYING BY 5 AND 9

The worksheets in the lesson provide practice with the multiplication facts. The children learn some properties of multiplication by 5 and 9.

MATERIALS

- Worksheets 40-47 (independent)
- Worksheets 48-50 (fast workers)

PROCEDURE

The children complete Worksheets 40 through 47 independently. Fast workers can do Worksheets 48, 49 and 50.

Here is another set of worksheets.

Everyone does Worksheets 40-47.

Fast workers can do Worksheets 48-50.

Worksheets 40 and 41 provide a review of most of the basic multiplication facts. If the children have trouble with any of the problems, suggest that they use arrays, repeated addition, a multiplication machine from Unit 25, or the multiplication charts on earlier worksheets in this unit.

On Worksheet 42 the children fill in a row and column on a multiplication chart with 9 as a factor. When 9 is a factor, the sum of the digits in the product must total 9. There is one exception to this rule. When one factor is 9 and the other factor is 0, the product is 0 and the digit does not total 9.

On Worksheets 43 and 44 the children discover that when one factor is 5 and the other factor is another whole number, the product always ends in either 0 or 5. Worksheet 44 also includes some review of the rule that when one factor is 9, the sum of the digits in the product equals 9 (unless the other factor is 0).

Worksheets 45 and 46 involve word problems and number lines that serve as models for multiplication.

Worksheets 48, 49 and 50 are for fast workers.

Worksheets 48 and 49 involve word problems that reinforce the similarity between multiplication and repeated addition.

On Worksheet 50 the children fill in products on a chart for problems with 0 as a factor. They work multiplication problems involving 0 as a factor.

60

Worksheet 40 Name ____________
Unit 27

Fill in the missing numbers in each sentence.

Remember: product = factor · factor

p = 8 · f	p = 7 · f	p = 3 · f	p = 4 · f
0 = 8 · 0	0 = 7 · 0	0 = 3 · 0	0 = 4 · 0
8 = 8 · 1	7 = 7 · 1	3 = 3 · 1	4 = 4 · 1
16 = 8 · 2	14 = 7 · 2	6 = 3 · 2	8 = 4 · 2
24 = 8 · 3	21 = 7 · 3	9 = 3 · 3	12 = 4 · 3
32 = 8 · 4	28 = 7 · 4	12 = 3 · 4	16 = 4 · 4
40 = 8 · 5	35 = 7 · 5	15 = 3 · 5	20 = 4 · 5
48 = 8 · 6	42 = 7 · 6	18 = 3 · 6	24 = 4 · 6
56 = 8 · 7	49 = 7 · 7	21 = 3 · 7	28 = 4 · 7
64 = 8 · 8	56 = 7 · 8	24 = 3 · 8	32 = 4 · 8
72 = 8 · 9	63 = 7 · 9	27 = 3 · 9	36 = 4 · 9
80 = 8 · T	7T = 7 · T	3T = 3 · T	4T = 4 · T

Worksheet 41 Name ____________
Unit 27

Fill in the missing numbers.

Remember: product = factor · factor

P = 5 · f	P = 9 · f	P = T · f	P = 6 · f
0 = 5 · 0	0 = 9 · 0	0 = T · 0	0 = 6 · 0
5 = 5 · 1	9 = 9 · 1	T = T · 1	6 = 6 · 1
10 = 5 · 2	18 = 9 · 2	2T = T · 2	12 = 6 · 2
15 = 5 · 3	27 = 9 · 3	3T = T · 3	18 = 6 · 3
20 = 5 · 4	36 = 9 · 4	4T = T · 4	24 = 6 · 4
25 = 5 · 5	45 = 9 · 5	5T = T · 5	30 = 6 · 5
30 = 5 · 6	54 = 9 · 6	6T = T · 6	36 = 6 · 6
35 = 5 · 7	63 = 9 · 7	7T = T · 7	42 = 6 · 7
40 = 5 · 8	72 = 9 · 8	8T = T · 8	48 = 6 · 8
45 = 5 · 9	81 = 9 · 9	9T = T · 9	54 = 6 · 9
5T = 5 · T	9T = 9 · T	10T = T · 10	6T = 6 · T

Worksheet 42 Name ____________
Unit 27

① Finish multiplying the factors in row 9 and column 9 in this multiplication chart.

② Now fill in these blanks. Add the digits.

	0	1	2	3	4	5	6	7	8	9
0										0
1										9
2										18
3										27
4										36
5										45
6										54
7										63
8										72
9			18	27	36	45	54	63	72	81

0
9
1 + 8 = 9
2 + 7 = 9
3 + 6 = 9
4 + 5 = 9
5 + 4 = 9
6 + 3 = 9
7 + 2 = 9
8 + 1 = 9

		1	2	3	4	5	6	7	8
		+8	+7	6	5	4	3	2	1
0	9	9	9	9	9	9	9	9	9

③ Fill in these blanks. Add the digits.

Worksheet 42 (continued)

④ When one factor in a multiplication problem is 9, what do you notice about the digits in the product?

Example: 5 · 9 = 45 4 + 5 = 9
9 · 3 = 27 2 + 7 = 9

When one factor in a multiplication problem is 9, the sum of the digits in the product equals 9.

⑤ Can you find a multiplication problem that has 9 as a factor, but the sum of the digits in the product does not equal 9? Write the problem here:

0 · 9 = 0

Did you try this? 0 · 9 = 0
What is the sum of the digits in the product? 0 There is no sum because there is only one digit.

⑥ When one factor is 9 and the other factor is a number other than 0, the product is 9 or the sum of the digits of the product equals 9.

⑦ Does 7 · 9 = 62? YES [✓ NO] 6 + 2 = 8
Does 9 · 4 = 35? YES [✓ NO] 3 + 5 = 8
Does 9 · 8 = 72? [✓ YES] NO 7 + 2 = 9

Worksheet 43 Name ____________
Unit 27

Finish the chart.

	0	1	2	3	4	5	6	7	8	9
0						0				
1						5				
2						10				
3						15				
4						20				
5	0	5	10	15	20	25	30	35	40	45
6						30				
7						35				
8						40				
9						45				

0 · 5 = 0
5 · 0 = 0
1 · 5 = 5
5 · 1 = 5

2 · 5 = 10
5 · 2 = 10
3 · 5 = 15
5 · 3 = 15

4 · 5 = 20
5 · 4 = 20
5 · 5 = 25
5 · 5 = 25

6 · 5 = 30
5 · 6 = 30
7 · 5 = 35
5 · 7 = 35

When one factor is 5 and the other factor is a whole number, the product always ends in 0 or 5.

Worksheet 44 Name ____________
Unit 27

Are these sentences true or false?

7 · 5 = 34 True [✓ False]
5 · 9 = 46 True [✓ False]
5 · 0 = 2 True [✓ False]
6 · 5 = 29 True [✓ False]
9 · 5 = 45 [✓ True] False
5 · 7 = 35 [✓ True] False

(Hint: does the product end in 0 or 5?)

Caution:
0 · 5 = 5 True [✓ False]

0 · 5 = 0

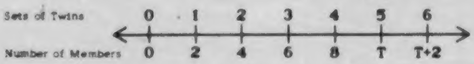
9 · 9 = 81 [✓ True] False
1 · 9 = 1 True [✓ False]
3 · 9 = 26 True [✓ False]

(Hint: do the digits in the product add up to 9?)

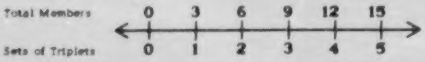
Caution:
0 · 9 = 0 [✓ True] False

(Hint: remember the exception to the rule.)

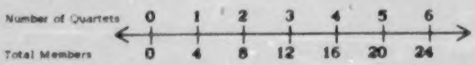
Worksheet 45 Name ____________
Unit 27



Every set of twins has 2 members.
In 5 sets of twins,
there are 5 · 2 or 10 members.

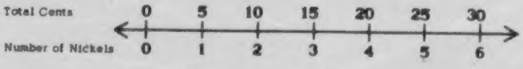


Every set of triplets has 3 members.
In 4 sets of triplets,
there are 4 · 3 or 12 members.

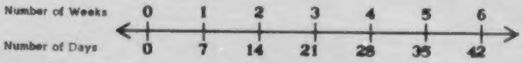


Every quartet has 4 members.
In 3 quartets,
there are 3 · 4 or 12 members.

Worksheet 46 Name ____________
Unit 27



Every nickel is worth 5 cents.
Three nickels are worth
3 · 5 or 15 cents.



Every week contains 7 days.
In 4 weeks,
there are 4 · 7 or 28 days.

In 2 weeks,
there are 2 · 7 or 14 days.

In 6 weeks,
there are 6 · 7 or 42 days.

Worksheet 47
Unit 27

Name ____________

Fill in the empty squares.

FACTORS

FACTORS

·	0	1	2	3	4	5	6	7	8	9
7	■	■	■	21	■	■	42	49	■	■
5	0	5	■	■	■	■	30	■	■	45
8	■	■	16	24	32	40	48	■	64	72
9	■	■	■	■	36	■	■	■	72	■
0	0	0	■	■	0	0	■	0	0	0
2	0	2	4	6	■	10	■	■	■	18
1	0	■	■	3	■	■	■	7	8	9
3	0	■	6	9	12	■	18	21	24	27
4	■	■	■	12	■	■	■	28	■	■
6	■	■	12	■	■	30	■	■	48	■

·	6	9	3
7	42	63	21
4	24	36	12
8	48	72	24

·	4	7	8
4	16	28	32
5	20	35	40
7	28	49	56

Fast Workers

Worksheet 48
Unit 27

Name ____________

1. A nickel is worth 5 cents.
   Six nickels are worth 5 + 5 + 5 + 5 + 5 + 5 or 30 cents.
   When I add six 5's, I get 6 · 5 or 30 cents.

2. Three nickels are worth 5 + 5 + 5 or 15 cents.
   When I add three 5's, I get 3 · 5 or 15 cents.

3. Seven nickels are worth 5 + 5 + 5 + 5 + 5 + 5 + 5 or 35 cents.
   When I add seven 5's, I get 7 · 5 or 35 cents.

Fast Workers

Worksheet 49
Unit 27

Name ____________

1. When I add nine 5's, I get 9 · 5 or 45
   Nine nickels are worth 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 or 45 cents.

2. When I add three 6's, I get 6 + 6 + 6 or 3 · 6.
   When I add four 7's, I get 7 + 7 + 7 + 7 or 4 · 7.
   When I add five 9's, I get 9 + 9 + 9 + 9 + 9 or 5 · 9.
   When I add X 4's, I get X · 4.
   When I add N 3's, I get N · 3.

3. I earn $2.00 per day.
   In 7 days, I earn 2 + 2 + 2 + 2 + 2 + 2 + 2 or 7 · 2 or $14.00.

Fast Workers

Worksheet 50
Unit 27

Name ____________

Finish the chart and answer the questions.

·	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0									
2	0									
3	0									
4	0									
5	0									
6	0									
7	0									
8	0									
9	0									

0 · any number = 0
any number · 0 = 0
0 · 1 = 1 · 0 = 0
0 · 2 = 2 · 0 = 0
0 · 3 = 3 · 0 = 0
0 · 4 = 4 · 0 = 0
$\frac{1}{2} \cdot 0 = 0 \cdot \frac{1}{2} = 0$
x · 0 = 0 · x = 0
0 · 23 = 23 · 0 = 0

WATCH OUT!

674,280 · 0 = 0

$0 \cdot 18\frac{1}{4} = 0$

$0 \cdot 83\frac{1}{2} = 0$

0 · 0 = 0

$\frac{23}{568} \cdot 0 = 0$

63 · 0 = 0

Section 2

Multiplication with Arrays

SECTION 2 MULTIPLICATION WITH ARRAYS

PURPOSE

In this section the students:

- Get further practice with addition, subtraction and multiplication problems.
- Learn to represent rectangular arrays as rectangles without the internal divisions.
- Partition a large rectangular array into smaller arrays and find the total number of elements in the original array by adding the number of elements in each smaller array.

COMMENTARY

In the lessons in this section the students will again be working alone or with partners on independent worksheets with a minimum of help from you. The worksheets have been divided to approximate one day's work per lesson. You may have to adjust this schedule according to the children's abilities.

Lesson 8 provides practice with multiplication facts using a circular multiplication slide rule. The commutative property of multiplication is reviewed with arrays and with multiplication problems.

In Lesson 9 the students draw rectangles to represent arrays, and then partition a rectangular array into two smaller arrays.

Lesson 10 includes arithmetic games using word problem cards. These cards provide practice with multiplication, addition and subtraction.

In Lesson 11 the students again partition rectangular arrays. This time they partition a large array into more than two smaller arrays. They find the total number of elements in the original array by adding the number of elements in each smaller array.

In Lesson 12 the students multiply with more than two factors. Multiplication is a binary operation, and the students learn to multiply two factors at a time when given a problem involving more than two factors. The worksheets also provide experience with multiplying multiples of 10 and 100.

Lesson 8: MULTIPLICATION PRACTICE

Independent worksheets review the commutative property of multiplication in the context of arrays. The children construct a circular multiplication slide rule and work some problems with it.

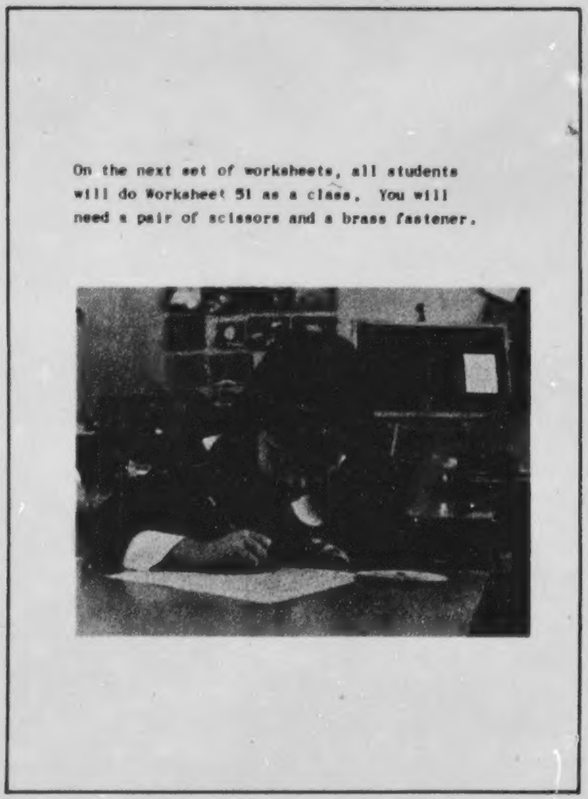
MATERIALS

- scissors for each child
- small brass fastener for each child
- Worksheet 51 (class)
- Worksheets 52-54 (independent)
- Worksheet 55 (fast workers)

PROCEDURE

Activity A

Have the children tear out Worksheet 51 and cut out the circular slide rule from it. A full-size copy of this worksheet is on the next page. After cutting out the two circles, each child assembles the slide rule by placing the smaller circle on top of the larger one and poking a small brass fastener through the centers. Tell the children to be sure the center of one circle lines up with the other before inserting the brass fastener. Also tell them not to poke too large a hole with the brass fastener. If the hole is too large the number scales on the two circles won't line up properly.

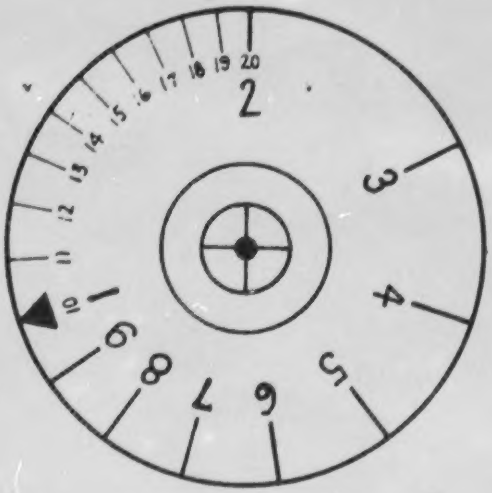


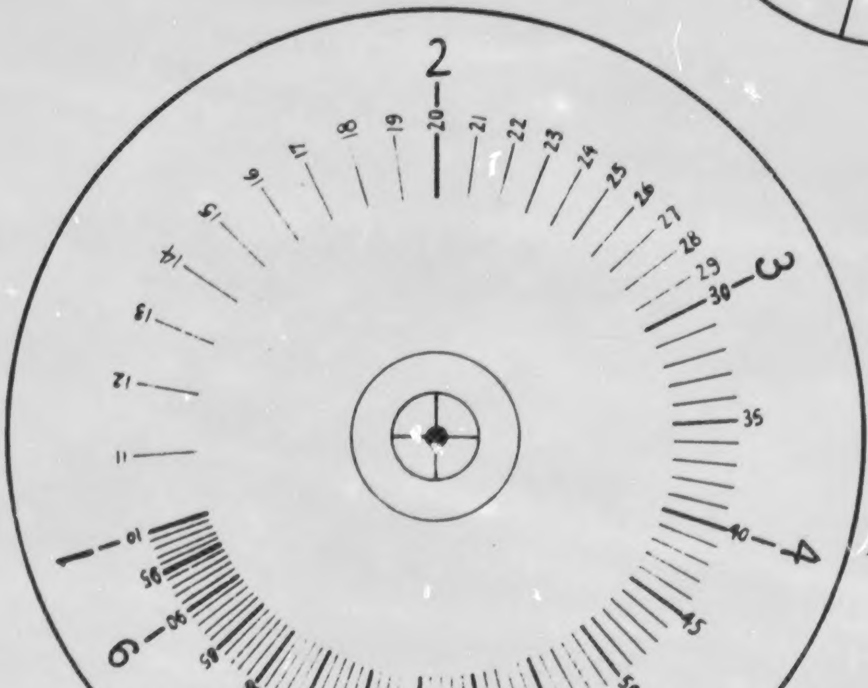
On the next set of worksheets, all students will do Worksheet 51 as a class. You will need a pair of scissors and a brass fastener.

Worksheet 51
Unit 27

Name ____________________

Circular Multiplication Slide Rule





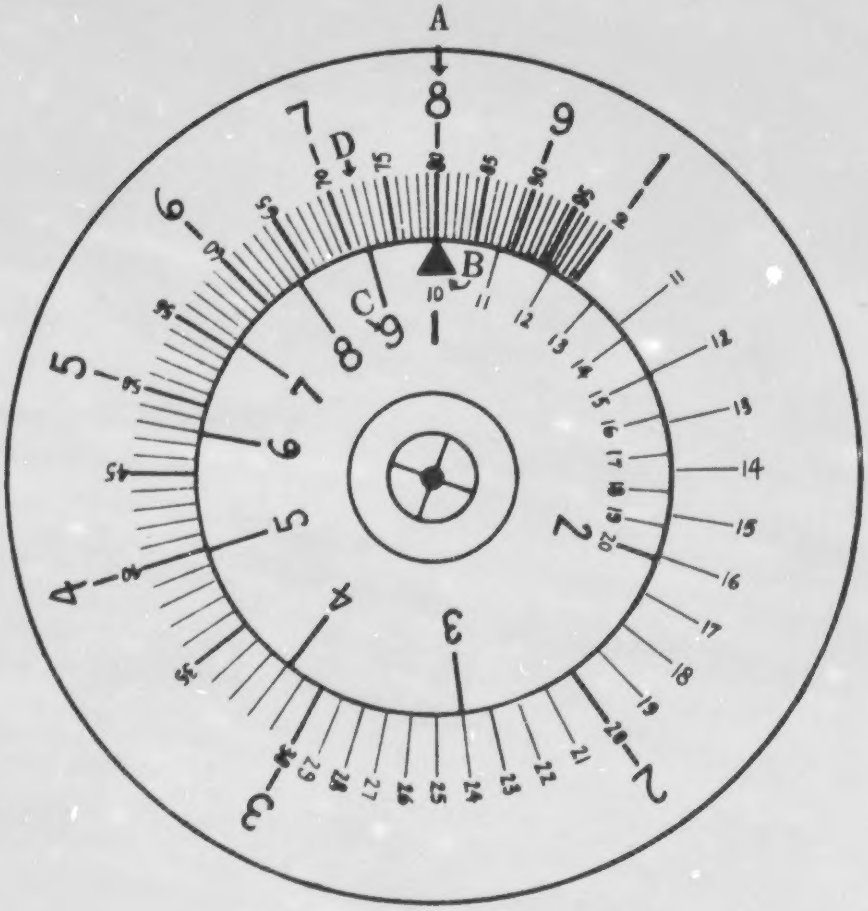
Here is how to use the slide rule to multiply. To multiply 8 · 9, find 8 on the outside numbers of the large circle. Turn the small circle until the arrow on it (by 10 and 1) points to the 8 on the large circle. Now find the 9 on the small circle. Follow it to the numbers on the large circle. The 9 points to the product, 72, on the large circle.

A. Find 8 on the large circle.

B. Turn the small circle until the arrow (▲) points at the 8.

C. Find 9 on the small circle.

D. Find the product, 72, on the large circle.



Have the children work a few multiplication problems while you check to see if they are using the slide rule properly. On most problems, the product will be the small number on the large circle. But with some problems, such as 2 · 3, the product is 6 and not 60. Single-digit products are read from the large numbers on the large circle. Give the children a problem where the product is a single-digit. Tell them that when their slide rule points to two numbers on the large circle (such as 2 and 20, 3 and 30, 4 and 40, etc.), they must think about that problem to decide which number is the product.

Using this slide rule the children can multiply the basic multiplication facts and several combinations of 1 through 9 (on the large circle) times 1 through 20 (on the small circle). If the children try to multiply 9 · 20, the slide rule will point to 18 on the large circle. The product is 180.

Activity B

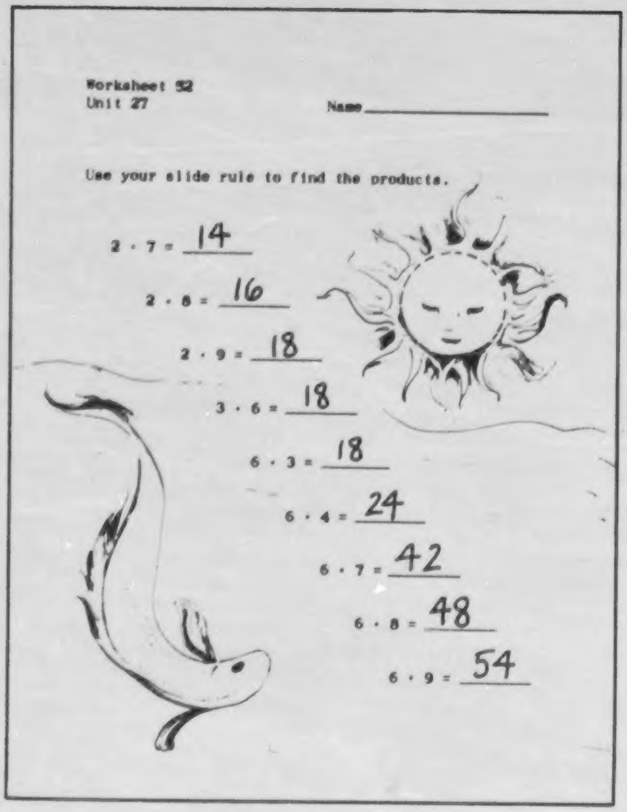
Have the children complete Worksheets 52 through 54 independently or with partners. Worksheet 55 is for fast workers.

On Worksheet 52 the students use their slide rule to find the products of multiplication problems. (Have the children save the slide rules to use whenever they need help with multiplication.)

Worksheet 53 reviews the commutative property of multiplication in the context of arrays. The children do not use their slide rules for this worksheet.

Worksheet 54 also reviews commutativity in multiplication problems. The students work problems such as $4 \cdot 7 = 7 \cdot 4$ and $AA \cdot BC = BC \cdot AA$. The children do not use their slide rules for this worksheet.

Worksheet 55 is for fast workers. It involves multiplication problems using the slide rule. One factor in each problem is a 2-digit number. There are two bonus problems: $6 \cdot 20$ and $9 \cdot 20$. Using the slide rule, the children will locate 12 and 18 as the products. The products, of course, are 120 and 180. Perhaps some children will realize that $6 \cdot 20$ could not equal 12 and $9 \cdot 20$ could not equal 18. If they ask you about this, tell them they are correct -- the products are not 12 and 18. Lead them to figure out the correct products.

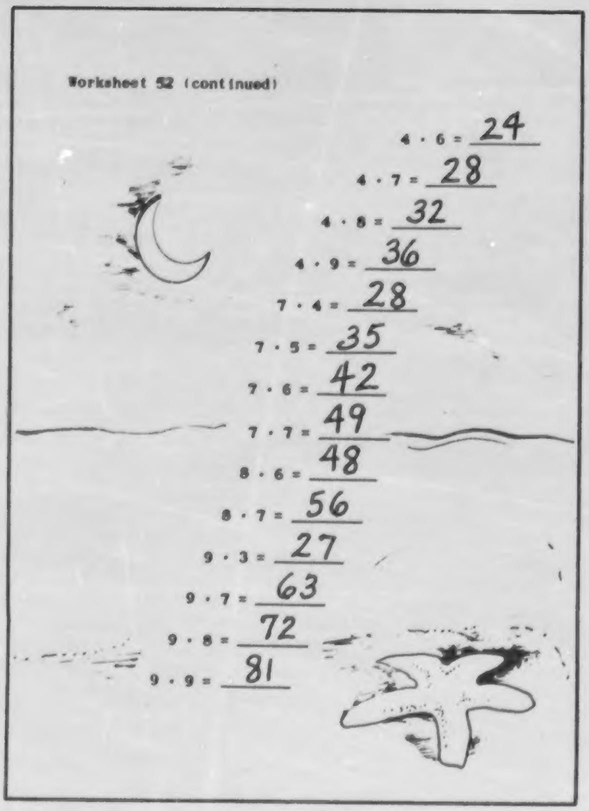


Worksheet 52
Unit 27

Name____________________

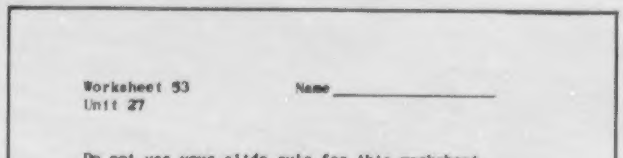
Use your slide rule to find the products.

2 · 7 = 14
2 · 8 = 16
2 · 9 = 18
3 · 6 = 18
6 · 3 = 18
6 · 4 = 24
6 · 7 = 42
6 · 8 = 48
6 · 9 = 54



Worksheet 52 (continued)

4 · 6 = 24
4 · 7 = 28
4 · 8 = 32
4 · 9 = 36
7 · 4 = 28
7 · 5 = 35
7 · 6 = 42
7 · 7 = 49
8 · 6 = 48
8 · 7 = 56
9 · 3 = 27
9 · 7 = 63
9 · 8 = 72
9 · 9 = 81



Worksheet 53
Unit 27

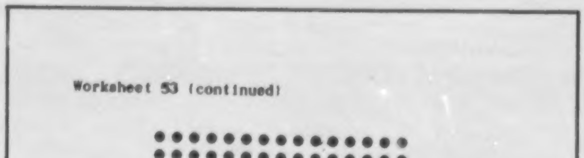
Name____________________

Do not use your slide rule for this worksheet.

Rows go across.
Columns go up and down.

Look at the array above.
Fill in these blanks.

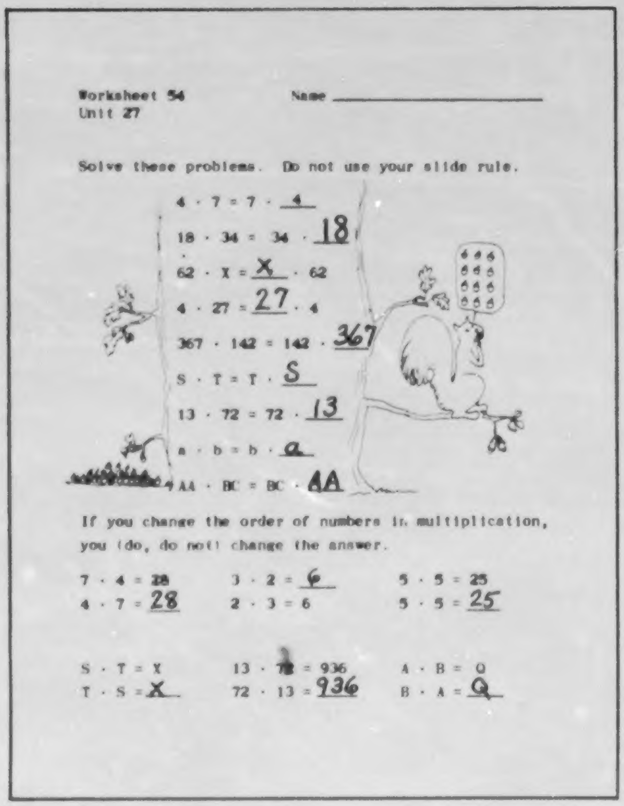
4 rows, 3 in a row, 4 · 3 altogether.
3 columns, 4 in a column, 3 · 4 altogether.
so 4 · 3 = 3 · 4



Worksheet 53 (continued)

8 rows, 15 in a row, 8 · 15 altogether.
15 columns, 8 in a column, 15 · 8 altogether.
so 8 · 15 = 15 · 8

6 columns, 7 in a column, 6 · 7 altogether.
7 rows, 6 in a row, 7 · 6 altogether.
so 6 · 7 = 7 · 6



Worksheet 54 Name ____________

Unit 27

Solve these problems. Do not use your slide rule.

$4 \cdot 7 = 7 \cdot \underline{4}$

$18 \cdot 34 = 34 \cdot \underline{18}$

$62 \cdot X = \underline{X} \cdot 62$

$4 \cdot 27 = \underline{27} \cdot 4$

$367 \cdot 142 = 142 \cdot \underline{367}$

$S \cdot T = T \cdot \underline{S}$

$13 \cdot 72 = 72 \cdot \underline{13}$

$a \cdot b = b \cdot \underline{a}$

$AA \cdot BC = BC \cdot \underline{AA}$

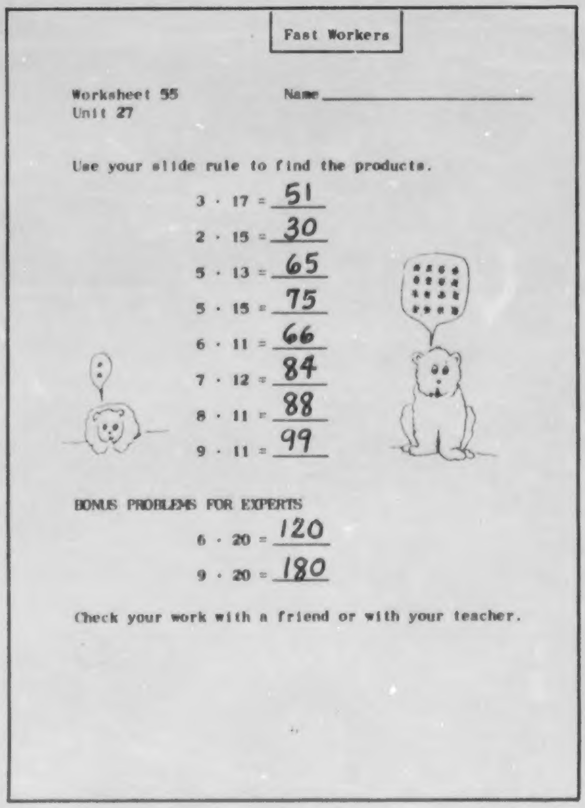
If you change the order of numbers in multiplication, you (do, do not) change the answer.

$7 \cdot 4 = 28$ $3 \cdot 2 = \underline{6}$ $5 \cdot 5 = 25$

$4 \cdot 7 = \underline{28}$ $2 \cdot 3 = 6$ $5 \cdot 5 = \underline{25}$

$S \cdot T = X$ $13 \cdot 72 = 936$ $A \cdot B = Q$

$T \cdot S = \underline{X}$ $72 \cdot 13 = \underline{936}$ $B \cdot A = \underline{Q}$



Fast Workers

Worksheet 55 Name ____________

Unit 27

Use your slide rule to find the products.

$3 \cdot 17 = \underline{51}$

$2 \cdot 15 = \underline{30}$

$5 \cdot 13 = \underline{65}$

$5 \cdot 15 = \underline{75}$

$6 \cdot 11 = \underline{66}$

$7 \cdot 12 = \underline{84}$

$8 \cdot 11 = \underline{88}$

$9 \cdot 11 = \underline{99}$

BONUS PROBLEMS FOR EXPERTS

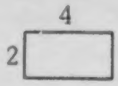
$6 \cdot 20 = \underline{120}$

$9 \cdot 20 = \underline{180}$

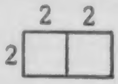
Check your work with a friend or with your teacher.

Lesson 9: PARTITIONING ARRAYS INTO TWO SMALLER ARRAYS

The children use the terms: addend, sum, factor and product. They represent rectangular arrays in a form without internal divisions:



The students partition rectangular arrays into two smaller rectangular arrays:



MATERIALS

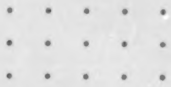
- Worksheets 56 and 57 (class discussion)
- Worksheets 58-61 (independent)

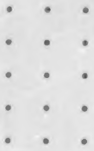
PROCEDURE

The children complete Worksheets 56 and 57 during the class discussion in Activity A. In Activity B, the students do Worksheets 58 through 61 independently or with partners.

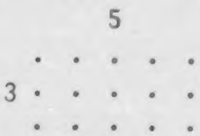
Activity A

Write a pair of factors such as 3,5 on the chalkboard and ask a student to draw a rectangular array to represent that pair of factors. He should draw one of the following:

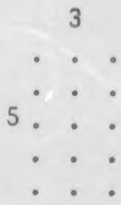




Label the array and have another student write the correct multiplication sentence near it.



or

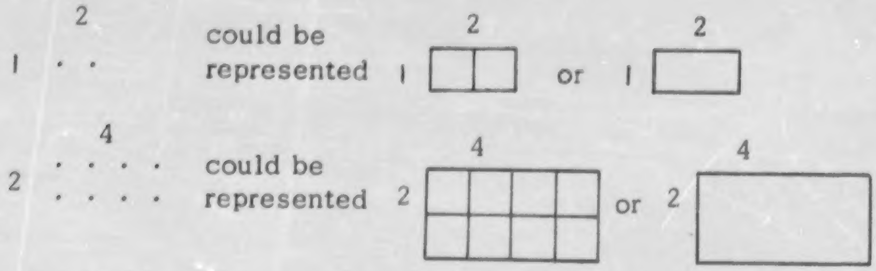


3 · 5 = 15

5 · 3 = 15

Leave this on the board.

Suggest to the children that they could save time in drawing arrays by using empty rectangles that have been properly labeled to represent the arrays. Put some examples like the following on the board.



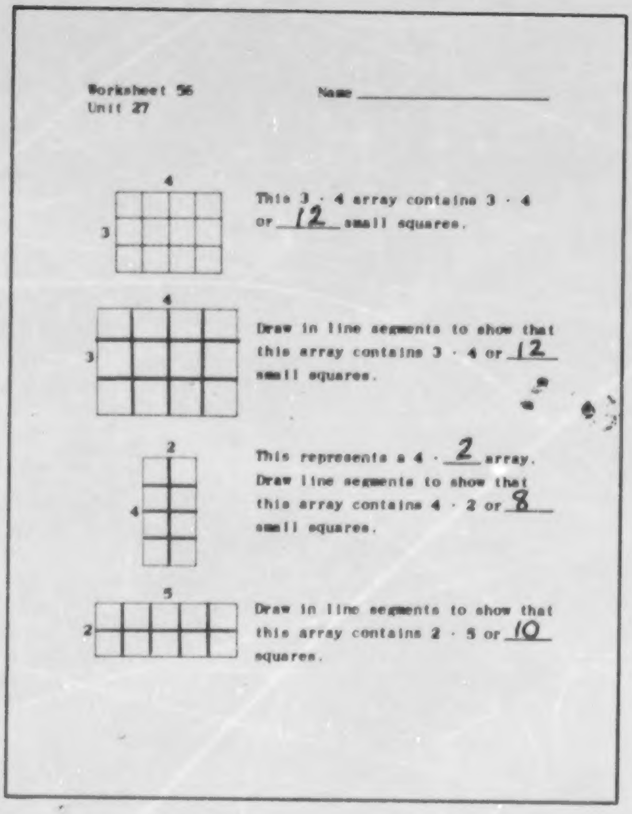
Refer back to the original array and ask how it would be redrawn as one large rectangle, labeled to represent a rectangular array. Have a child redraw it, label the parts, and write a multiplication sentence to describe it.

In the next lesson you will be working with arrays.

All students will do Worksheets 56 and 57 together.

Then you will do Worksheets 58-61 alone or with a partner.

Give the children a few mintues to complete Worksheets 56 and 57. Discuss the worksheets as a group when everyone has completed them.



Worksheet 56
Unit 27

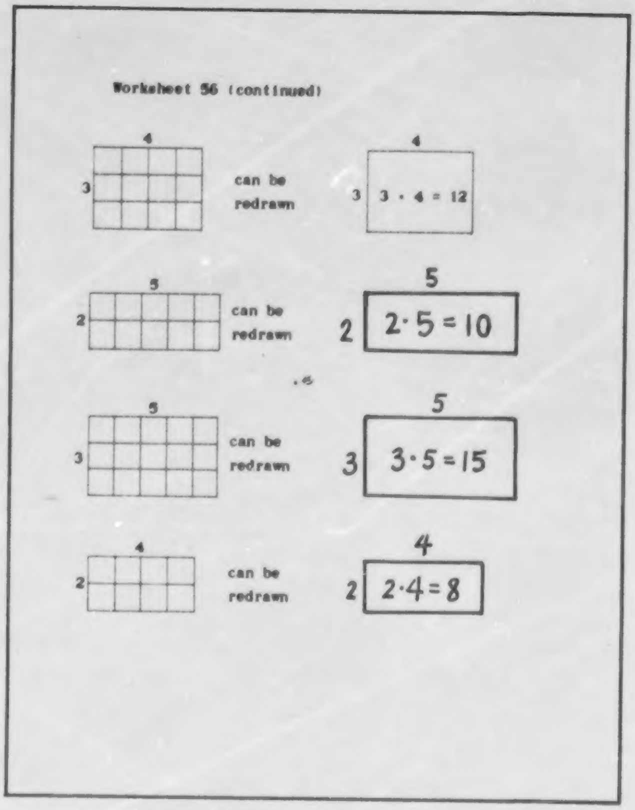
Name ____________

This 3 · 4 array contains 3 · 4 or 12 small squares.

Draw in line segments to show that this array contains 3 · 4 or 12 small squares.

This represents a 4 · 2 array. Draw line segments to show that this array contains 4 · 2 or 8 small squares.

Draw in line segments to show that this array contains 2 · 5 or 10 squares.



Worksheet 56 (continued)

4
3

can be redrawn

4
3

3 · 4 = 12

5
2

can be redrawn

5
2

2·5=10

5
3

can be redrawn

5
3

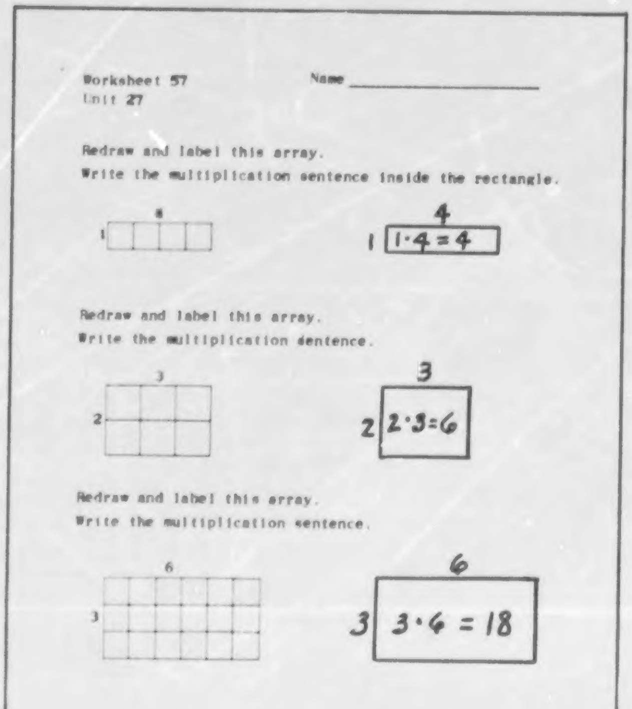
3·5=15

4
2

can be redrawn

4
2

2·4=8



Worksheet 57
Unit 27

Name ____________

Redraw and label this array.
Write the multiplication sentence inside the rectangle.

4
1
4
1

1·4=4

Redraw and label this array.
Write the multiplication sentence.

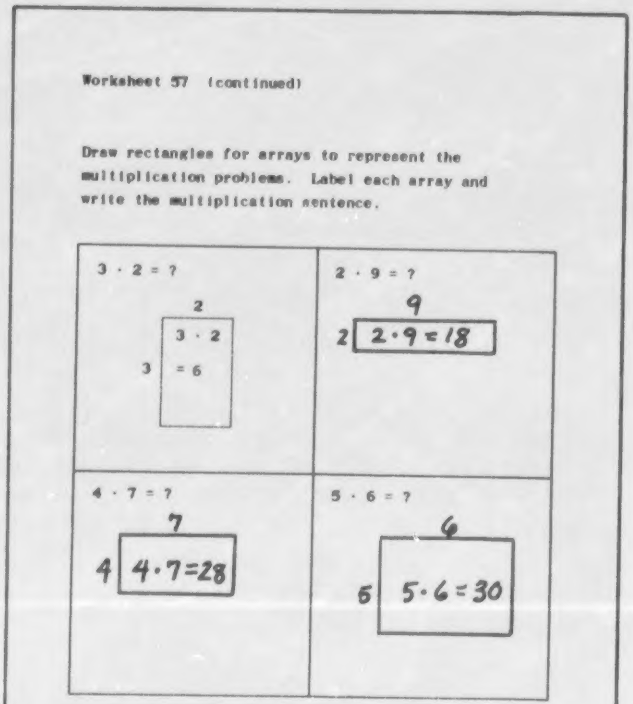
3
2
3
2

2·3=6

Redraw and label this array.
Write the multiplication sentence.

6
3
6
3

3·6 = 18



Worksheet 57 (continued)

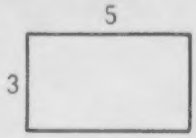
Draw rectangles for arrays to represent the multiplication problems. Label each array and write the multiplication sentence.

3 · 2 = ? 2 3 3 · 2 = 6	2 · 9 = ? 9 2 2·9=18
4 · 7 = ? 7 4 4·7=28	5 · 6 = ? 6 5 5·6=30

Activity B

Have the children complete Worksheet 58 as a review of addends, sums, factors and products, before they begin this activity.

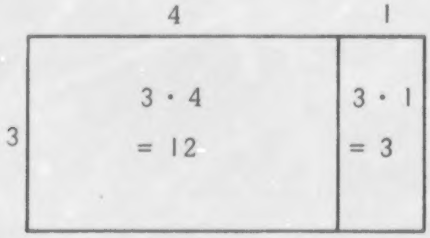
Draw and label this array on the chalkboard. Then ask the children if they can partition the array into two smaller rectangular arrays.



By partition, we mean to "break it down" into smaller arrays. You do not need to give the children any special definition of "partition." They should be able to pick up the meaning by the way you use it in sentences.

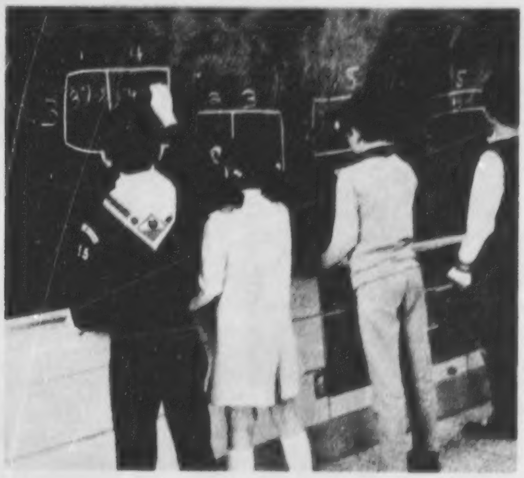
Let the children suggest ways to partition the array, and then have a student draw his solution on the chalkboard. He should label the parts of the array and write the correct multiplication sentences.

This is one example of how the array could be partitioned.

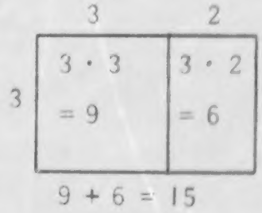


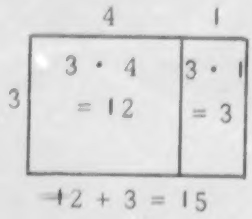
Have the students turn to Worksheet 59 and give them five minutes to draw partitions of the 3 x 5 array into two smaller rectangular arrays. They should draw all the possible partitions they can think of. You may want to have them work with partners.

When the children are finished, let individual children come to the chalkboard and write their solutions. If some children did not find all the possible solutions, have them copy those from the chalkboard onto their worksheets.



Ask the students to determine the total number of elements in each partitioned solution of the original 3 x 5 array. One way to find the total number of elements is to add the products of the multiplication sentences represented by the two smaller arrays.





The students should notice that even when the original array was partitioned into several different variations of two smaller arrays, the total number of elements in the array, 15, remains the same.

Have the students work with partners to complete Worksheets 60 and 61. They should partition these arrays using the procedure developed in this activity. On Worksheet 61 the children also find the total number of elements in each solution.

Worksheet 58 Name ____________
Unit 27

3 + 2 = 5 In this addition sentence,
the addends are 3 and 2.
The sum is 5.

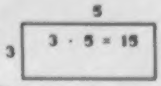
3 · 2 = 6 In this multiplication sentence,
the factors are 3 and 2.
The product is 6.

Addends	Sum
3 , 5	8
4 , 9	13
6 , 9	15
7 , 3	10
8 , 4	12
9 , 6	15
10,10	20
7 , 7	14
6 , 4	10
4 , 7	11
3 , 3	6
12, 0	12

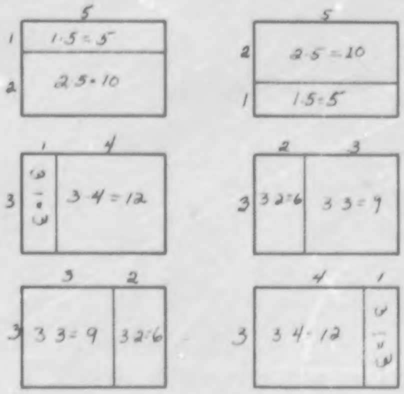
Factors	Product
3 , 5	15
4 , 9	36
6 , 9	54
7 , 3	21
8 , 4	32
9 , 6	54
10,10	100
7 , 7	49
6 , 4	24
4 , 7	28
3 , 3	9
12, 0	0

Worksheet 59 Name ____________
Unit 27

Draw all the possible partitions for this array:

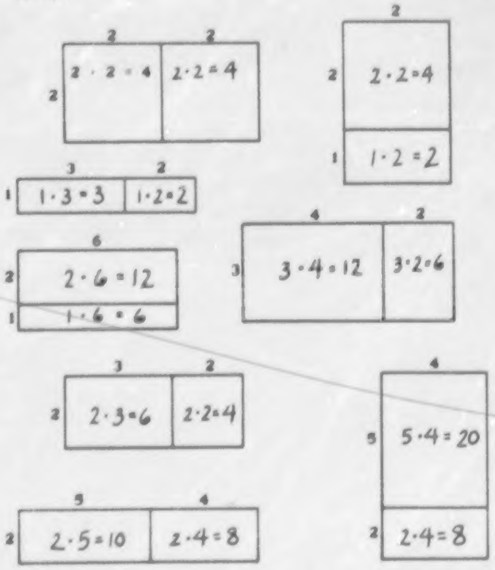


Write a multiplication sentence in each array.



Worksheet 60 Name ____________
Unit 27

Write multiplication sentences in the boxes of each array.

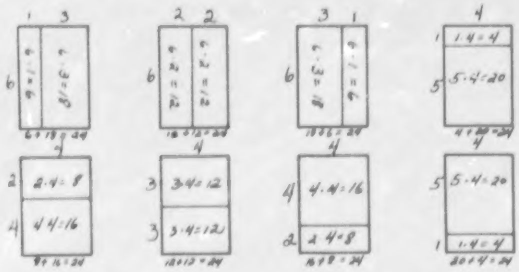


Worksheet 61 Name ____________
Unit 27

Partition this array into smaller arrays.

Draw as many solutions as you can.

Write multiplication sentences.



Now find the total number of elements in each solution. Hint: Add the products of both multiplication sentences in each solution.

Lesson 10: WORD PROBLEM GAMES

Word problem cards are used in this lesson to provide practice with multiplication, addition and subtraction. The numbers in the problems have been omitted so the problems can be varied to suit the needs of different children. The game should be introduced in this lesson, and then played whenever the children have free time.

MATERIALS

- Worksheets 62-68
- dice numbered 0-5 and 4-9 or spinner and discs
- scissors, paper and pencils

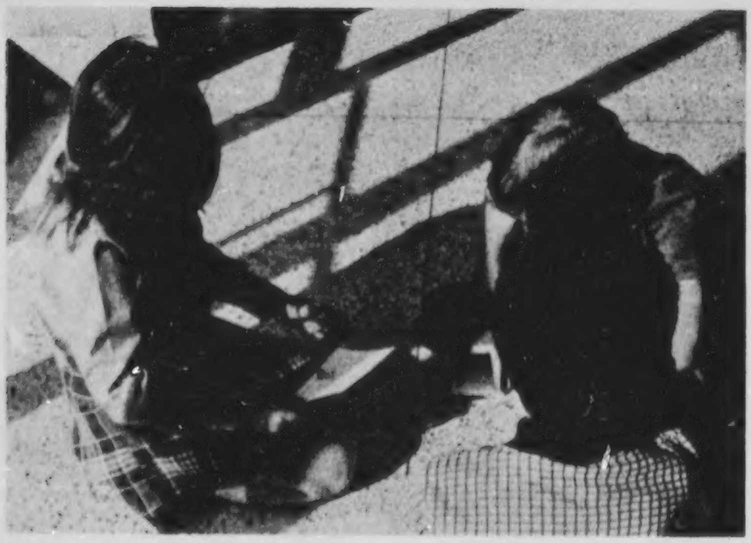
PROCEDURE

The arithmetic word problem cards are on Worksheets 62 through 67. Each deck consists of 36 word problems involving multiplication, addition, and subtraction. Have the children tear out the worksheets and cut up the cards. Worksheet 68 is a page of blank cards for each child to make up his own problems later, after the children have played the game a few times. Each child may wish to put his initials on his cards and store them in an envelope when not in use.

The children can generate numbers for the word problems in several different ways. Here are some suggestions.

1. Each child prepares about twenty slips of paper by marking each one with a numeral from 1 to 10. (Later, larger numbers can be used.) These are placed in a box. Then the numbers for a particular problem are selected by drawing two of the slips.

2. Make up lists of numeral pairs. This way you can control the combinations and provide numerals appropriate to the



skills of different pairs of children. The numbers for a problem are taken in sequence from one of these lists.

3. Use the game spinners and mark them with appropriate numbers. Each child in a pair spins once to obtain the two numerals for a problem.

4. Throw a pair of dice to obtain the numerals.

5. Let the children choose their own numbers.

To introduce the game, have the children work with partners. Each pair uses one deck of cards. One student shuffles the cards and puts the deck face down in front of them. The other student turns over the top card. Then both students read the problem and decide if it is a multiplication, addition, or subtraction problem. Then one student generates two numbers in one of the ways suggested above. (If a card involves a subtraction problem, they must use the numbers in a certain order. On multiplication and addition problems, they should use the numbers in the order generated.) Using the numbers they generated, each student in the pair writes the appropriate arithmetic sentence. They compare their answers. Then the students turn over the next card and follow the same procedure. Have each pair go through the whole deck in this manner. On the next page are three examples of what the children should do with the cards.

card:

numbers generated and sentence for each problem:

I feed the cat next door when his family is away. The cat eats ______ cans of food a day. If the family is gone ______ days, I will have to give him ______ cans of food.

3 and 7

3 · 7 = 21

I bought ______ pounds of sugar and ______ pounds of flour at the store. I could hardly carry it home because altogether it weighed ______ pounds.

5 and 9

5 + 9 = 14

My father gave my mother ______ roses. The cat ate ______ of them, [illegible] now there's only ______ left. My mother wants to get rid of that cat.

4 and 8

8 - 4 = 4

When the children understand the word problems and how to write an arithmetic sentence from a problem, have them play some of the following variations of the game. The children can also make up some of their own problems on Worksheet 68. They should make two problems each involving multiplication, addition and subtraction.

Variations

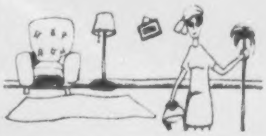
1. The children can play with partners, taking turns drawing a card, generating the numbers, writing the arithmetic sentence, and solving the problem. The child who gets the most correct, wins the game.
2. Several children can play. One child at a time draws a card, generates numbers, writes the sentence and solves that problem. The child who gets the most correct, wins the game.
3. Partners or several children can play with the rule that they cannot write down the sentence, but must do the computing in their heads.
4. Several children can play only with those cards each child made up himself.
5. The children can play any of the above variations using larger numbers.
6. The children should think of some of their own variations, and explain these to other members of the class.

In the next lesson you will play an arithmetic game with word problem cards. Have fun!

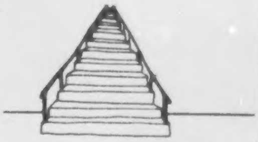
(62)

I planted ______carrot seeds in each row. I planted ______ rows. If all of the seeds grow, I will have ______ carrot plants in my garden.

There are ______rooms in our house. If my mother cleans the whole house ______ times this week, she will have cleaned ______ rooms.

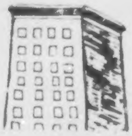


There are______ steps between floors in our apartment. If you walk up______ floors, you'll have to climb______ steps.



I like to cut the grass. I can do ______ rows before I rest. If I do ______ groups of rows, how many do I cut altogether? ______.

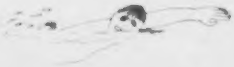
There are ______apartments on each floor. If there are ______floors in the building, there are ______apartments altogether.



I ride my bike ______ blocks a day. In ______ days I ride ______ blocks.

(63)

I swim ______times a week in summer. In______weeks I go swimming______ times.



Hamsters have ______babies at one time. If a mother hamster has babies ______times a year, she will have ______babies that year.

I read ______pages from my storybook each day. In ______days I read ______ pages.

There are______ buildings on our block. If______ people live in each building, how many people live on our block? ______.

I feed the cat next door when his family is away. The cat eats______cans of food a day. If the family is gone ______ days, I will have to give him______cans of food.

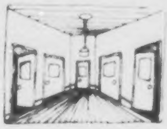
There are______ marbles in a box. If I get ______boxes of marbles for my birthday, how many marbles will I get? ______.

(64)

It rained______days this week. If the weather stays like this, in______ weeks we will have______rainy days.

The dog had______puppies. Each puppy has______spots on it. Altogether there are ______ spots on the puppies.

Each classroom has ______ lights in it. There are______classrooms on the hall. How many lights are there altogether? ______.



Each day I eat ______cookies for a snack. If I do this for ______days, I will eat ______cookies altogether.

Every page has ______problems on it. If we do ______pages, we will work ______problems.

Each time we play baseball, I get to bat______times. When we play ______ times, I bat______times.

(65)

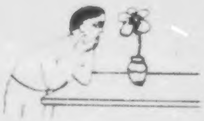
We take the dog for a walk ______ times every day. In ______ days the dog goes for ______ walks.

______ children sit in each row. There are ______ rows. How many children are there altogether? ______.

I can do ______ easy problems in ______ minute. If I work for ______ minutes I can do ______ problems.

Each team has ______ players. There are ______ teams. How many players are there altogether? ______

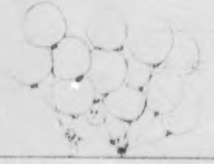
This flower has ______ petals. If I pick ______ flowers, I will have ______ petals altogether.



There are ______ pieces of candy in each package. Let's buy ______ packages. Then we will have ______ pieces of candy.

(66)

We had a birthday party for my little brother. I blew up ______ balloons and my big brother blew up ______ balloons. We have to get rid of all ______ balloons because our house is too crowded.



When we go to the lake for a week-end, I have to bring ______ pairs of pants and ______ shirts because I always fall in the lake. My mother has to find a suitcase that's big enough to hold ______ pieces of clothing.

I bought ______ pounds of sugar and ______ pounds of flour at the store. I could hardly carry it home because altogether it weighed ______ pounds.

I had a dream that I found ______ chocolate chip cookies and ______ peanut butter cookies in the cookie jar. My mother wasn't home, so I ate all ______ of them. Boy was I sick! Good thing it was only a dream!

To get to school from my house, I walk ______ blocks up Oak Street, turn right, and then go ______ blocks down Washington Avenue. I walk ______ blocks to school.

My library at home is getting so large, I need a new book case. I got ______ books for Christmas and ______ more for my birthday. I don't know where to put these ______ new books.

(67)

We studied our MINNEMAST for ______ days. I was at home sick with the flu for ______ days, so I only studied MINNEMAST for ______ days.

______ girls and boys were investigating some properties of minerals. ______ boys left, so there were only ______ people experimenting with the mineral samples.

If I had ______ cents, I'd go to the store and spend ______ cents on ice cream. I don't know what I'd do with the other ______ cents.

My father gave my mother ______ roses. The cat ate ______ of them, so now there's only ______ left. My mother wants to get rid of that cat.

I have to bring ______ pictures to school. So far I've only found ______ pictures, so I'll have to keep looking until I find ______ more.

There are ______ rides I'd like to go on at the carnival, but my mother said I can only ride on ______ of them. That means I can't ride on ______.

(68)

Make some more cards for your deck. Make 2 multiplication, 2 addition, and 2 subtraction word problems.

Lesson 11: PARTITIONING ARRAYS INTO SEVERAL SMALLER ARRAYS

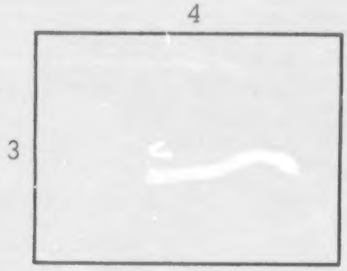
During a class discussion the students partition a rectangular array into as many smaller rectangular arrays as possible and then write a multiplication sentence for each smaller array. They find the total number of elements in the original array by adding the number of elements in each smaller array.

MATERIALS

- Worksheet 69 (class discussion)

PROCEDURE

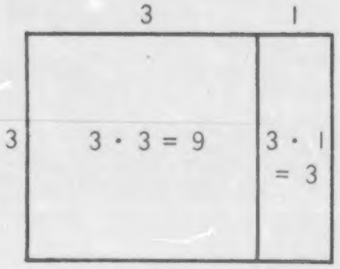
Draw this rectangular array on the chalkboard, making it large enough for all the students to see.

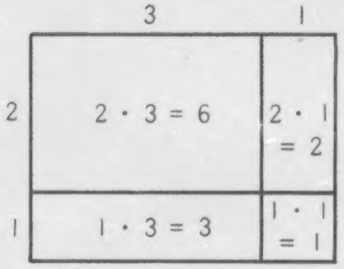


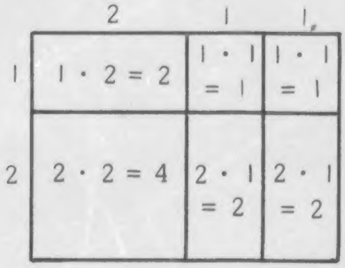
Then say:

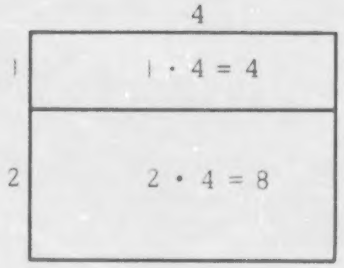
> PARTITION THIS RECTANGULAR ARRAY INTO MORE THAN ONE SMALLER RECTANGULAR ARRAY. LABEL EACH SMALLER ARRAY WITH A MULTIPLICATION SENTENCE.

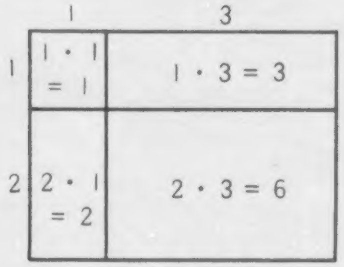
Let the students propose as many solutions as they can think of. They will probably suggest solutions that involve partitioning it into two smaller arrays. If no one remembers that you said "more than one smaller rectangular array," repeat the question. Lead the children to realize that "more than one" does not limit them to only two. Ask several volunteers to come to the board and draw their solutions. Each child should redraw the original array and partition it with his solution. He should also write a multiplication sentence in each smaller array. Some solutions are shown on the next page.

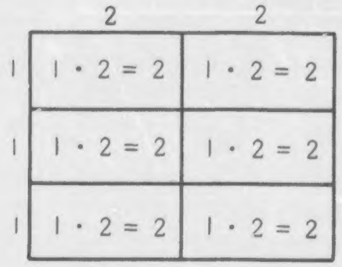




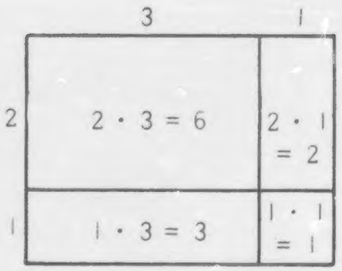




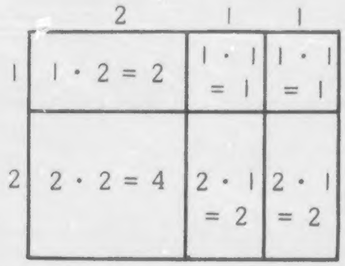




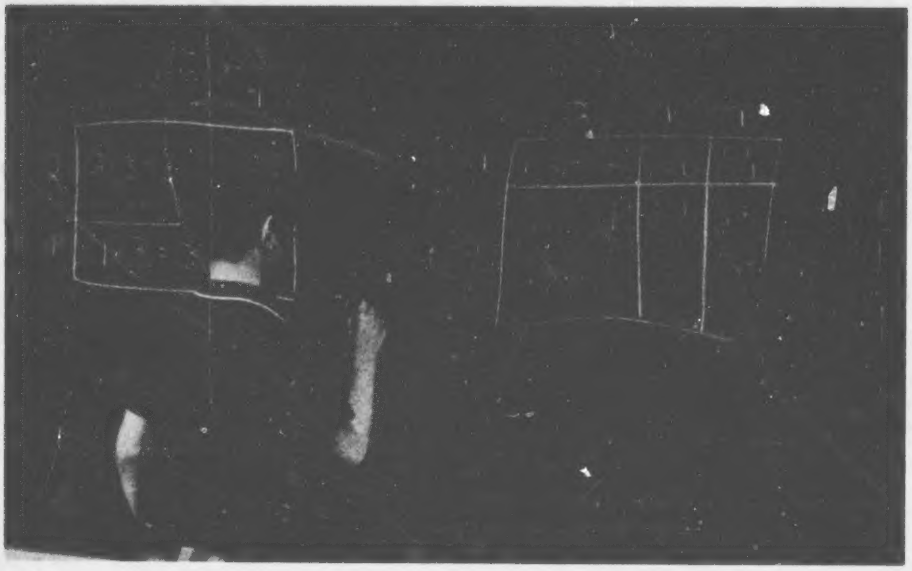
After each solution has been drawn and labeled, ask other volunteers to find the total number of elements in each original array by adding the number of elements in each smaller array.



$$\begin{array}{r} 6 \\ 2 \\ 3 \\ \underline{1} \\ 12 \end{array}$$



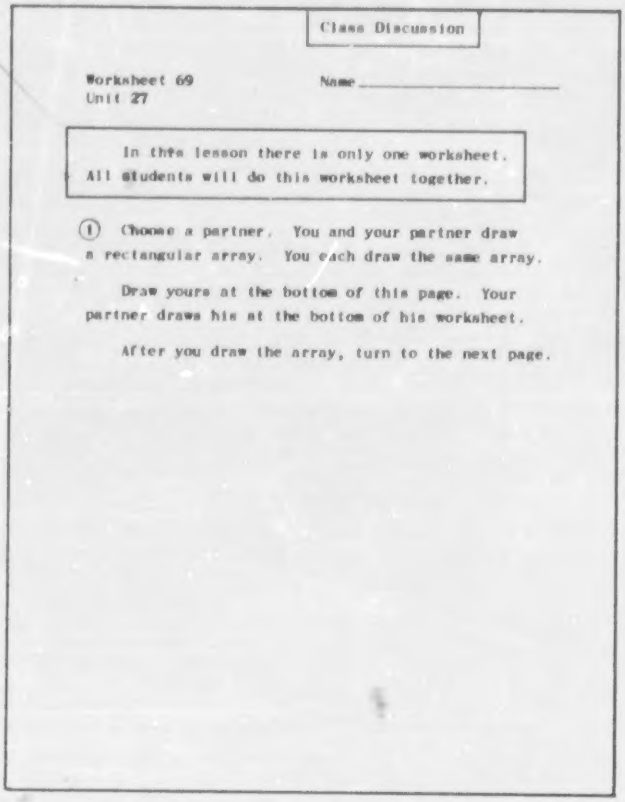
$$\begin{array}{r} 2 \\ 1 \\ 1 \\ 4 \\ 2 \\ \underline{2} \\ 12 \end{array}$$



When the students have discussed all the solutions they have found to the array problem on the board, have them pair off and turn to Worksheet 69.

Each pair draws a rectangular array. Then they draw as many partitions as they can for that array. They label each part and write a multiplication sentence in each smaller array. They add the elements of each smaller array to find the total number of elements in the original array.

Give the children 10 or 15 minutes to do this and then discuss their work. Some students may want to draw the original array and a couple of solutions to it on the chalkboard. The students should realize that the total number of elements in each solution is the same because the original array for each solution is the same.



Class Discussion

Worksheet 69
Unit 27

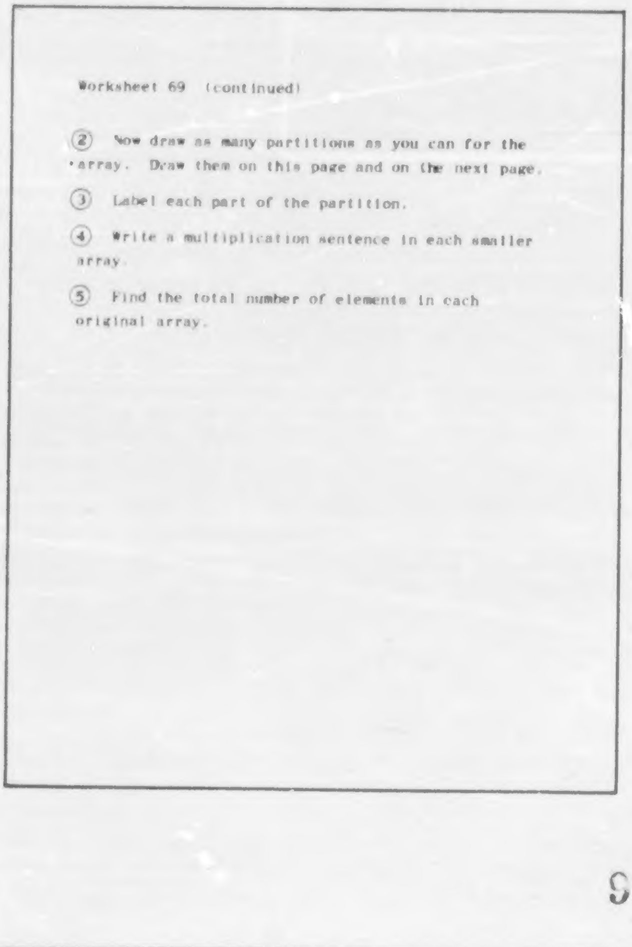
Name ____________________

In this lesson there is only one worksheet. All students will do this worksheet together.

① Choose a partner. You and your partner draw a rectangular array. You each draw the same array.

Draw yours at the bottom of this page. Your partner draws his at the bottom of his worksheet.

After you draw the array, turn to the next page.



Worksheet 69 (continued)

② Now draw as many partitions as you can for the array. Draw them on this page and on the next page.

③ Label each part of the partition.

④ Write a multiplication sentence in each smaller array.

⑤ Find the total number of elements in each original array.

Worksheet 69 (continued)

Lesson 12: MULTIPLYING WITH MORE THAN TWO FACTORS

Students multiply more than two factors to illustrate the associative property of multiplication. The independent worksheets provide practice with the multiplication facts and also with multiplying multiples of 10, 100, etc. by single-digit numbers.

MATERIALS

- Worksheets 70 - 75 (independent)

PROCEDURE

Have the students complete Worksheets 70 through 75 independently or with partners.

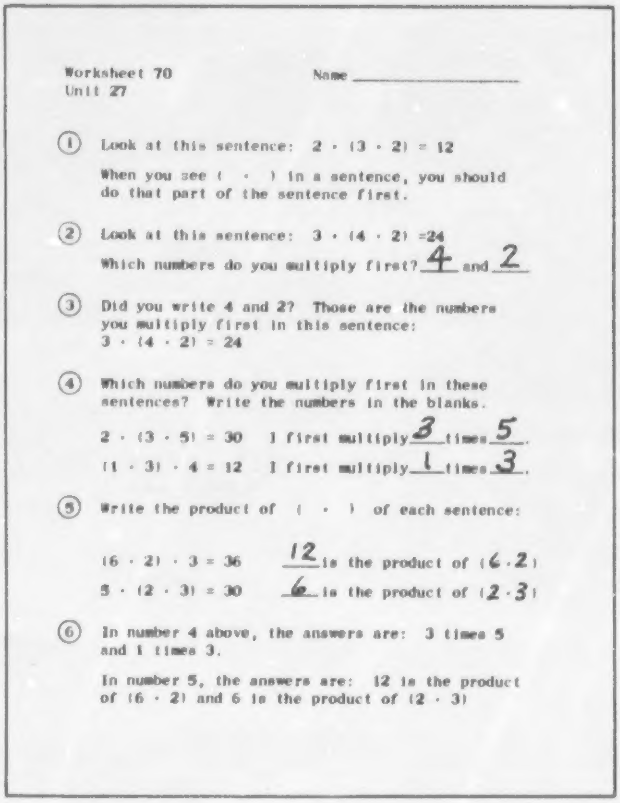
Multiplication is a binary operation. This means that only two members of a multiplication sentence can be multiplied at any one time. In this problem, $3 \cdot 2 \cdot 4$, you multiply $3 \cdot 2$ (equals 6), and then multiply $6 \cdot 4$ to get the product of the three factors, 24. What happens if you first multiply $2 \cdot 4$? You get 8, and then multiply $3 \cdot 8$ to get the product of the three factors, 24. In both cases, the order of the factors remained. In one case, the factor 2 was used in a binary operation with 3. In the other case the 2 was used with the 4. This difference is illustrated with the following sentences.

$$(3 \cdot 2) \cdot 4 = 24 \qquad 3 \cdot (2 \cdot 4) = 24$$

This property of multiplication is known as the associative property: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. The associative property states that if you regroup or reassociate the numbers in a multiplication sentence, you still get the same product.

On Worksheet 70 the students are introduced to the use of parentheses in a multiplication sentence. They are told to multiply the part of the sentence in parentheses first.

In the next lesson you will do Worksheets 70-75 alone or with a partner.



Worksheet 70
Unit 27

Name ____________

① Look at this sentence: 2 · (3 · 2) = 12

When you see (·) in a sentence, you should do that part of the sentence first.

② Look at this sentence: 3 · (4 · 2) =24

Which numbers do you multiply first? 4 and 2

③ Did you write 4 and 2? Those are the numbers you multiply first in this sentence:
3 · (4 · 2) = 24

④ Which numbers do you multiply first in these sentences? Write the numbers in the blanks.

2 · (3 · 5) = 30 I first multiply 3 times 5.
(1 · 3) · 4 = 12 I first multiply 1 times 3.

⑤ Write the product of (·) of each sentence:

(6 · 2) · 3 = 36 12 is the product of (6 · 2)
5 · (2 · 3) = 30 6 is the product of (2 · 3)

⑥ In number 4 above, the answers are: 3 times 5 and 1 times 3.

In number 5, the answers are: 12 is the product of (6 · 2) and 6 is the product of (2 · 3)

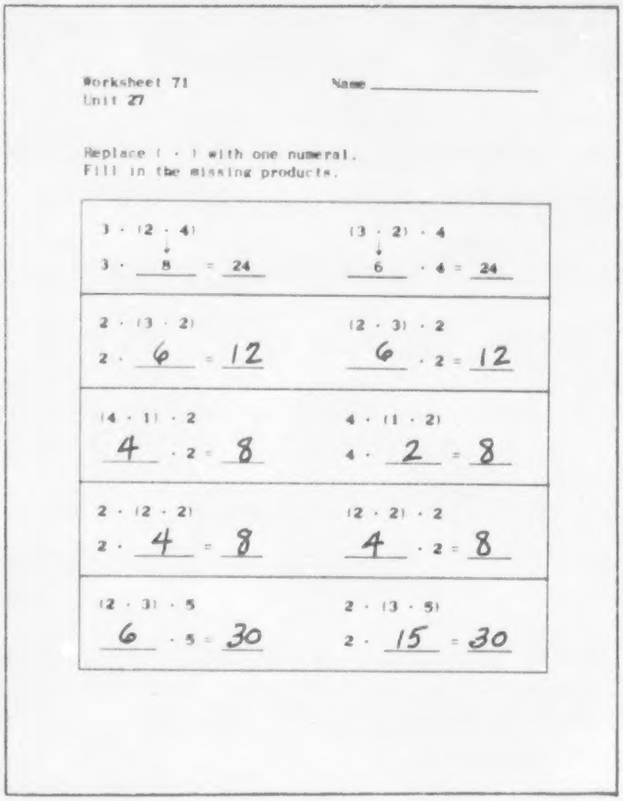
On Worksheet 71 the students study the associative property of multiplication by replacing (·) in a multiplication sentence with a single number.

On Worksheet 72 the students multiply (a · b) · c and a · (b · c). The values of a, b and c are given. Students who have difficulty may want to assign numerals to the a · b · c parts of each sentence.

On Worksheet 73 the students take a factor, such as 20, and rename it as a pair of factors, 2 · 10. Then a pair of factors, such as 3 · 20, becomes 3 · (2 · 10). The students also work problems such as 3 · 2 = 6 and 3 · 20 = 60 without renaming the 20 as (2 · 10).

Worksheet 74 gives more practice with the associative property by replacing the (·) part of a sentence with a single number.

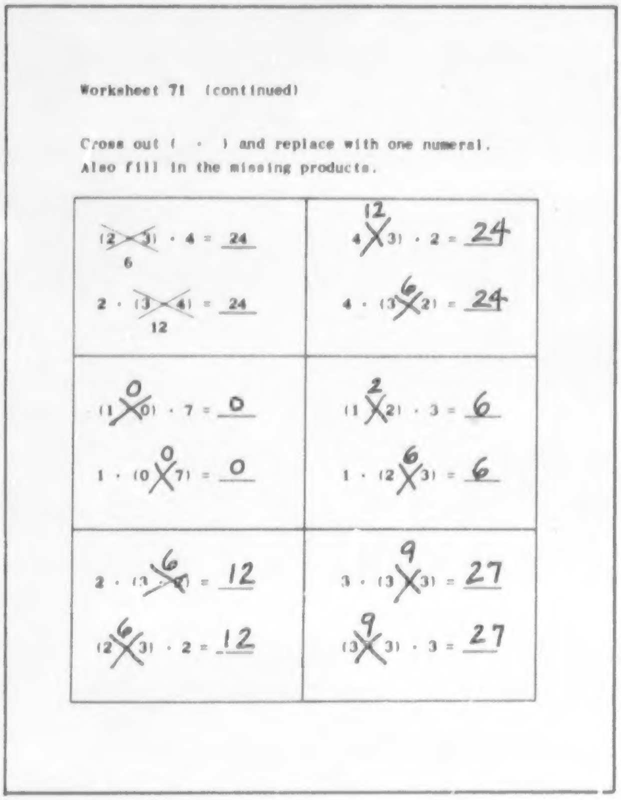
Worksheet 75 reviews the properties of multiplying by 0 and 1 and the commutative property of multiplication: a · b = b · a.



Worksheet 71 Name ____________________
Unit 27

Replace (·) with one numeral.
Fill in the missing products.

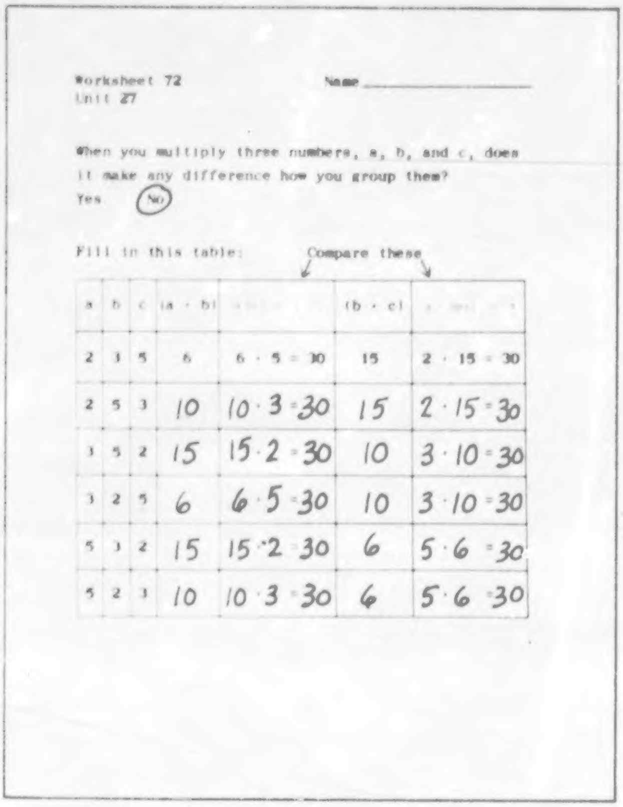
$3 \cdot (2 \cdot 4)$ $3 \cdot \underline{8} = \underline{24}$	$(3 \cdot 2) \cdot 4$ $\underline{6} \cdot 4 = \underline{24}$
$2 \cdot (3 \cdot 2)$ $2 \cdot \underline{6} = \underline{12}$	$(2 \cdot 3) \cdot 2$ $\underline{6} \cdot 2 = \underline{12}$
$(4 \cdot 1) \cdot 2$ $\underline{4} \cdot 2 = \underline{8}$	$4 \cdot (1 \cdot 2)$ $4 \cdot \underline{2} = \underline{8}$
$2 \cdot (2 \cdot 2)$ $2 \cdot \underline{4} = \underline{8}$	$(2 \cdot 2) \cdot 2$ $\underline{4} \cdot 2 = \underline{8}$
$(2 \cdot 3) \cdot 5$ $\underline{6} \cdot 5 = \underline{30}$	$2 \cdot (3 \cdot 5)$ $2 \cdot \underline{15} = \underline{30}$



Worksheet 71 (continued)

Cross out (·) and replace with one numeral.
Also fill in the missing products.

$\underset{6}{\cancel{(2 \cdot 3)}} \cdot 4 = \underline{24}$ $2 \cdot \underset{12}{\cancel{(3 \cdot 4)}} = \underline{24}$	$(4 \overset{12}{\cancel{\cdot}} 3) \cdot 2 = \underline{24}$ $4 \cdot (3 \overset{6}{\cancel{\cdot}} 2) = \underline{24}$
$(1 \overset{0}{\cancel{\cdot}} 0) \cdot 7 = \underline{0}$ $1 \cdot (0 \overset{0}{\cancel{\cdot}} 7) = \underline{0}$	$(1 \overset{2}{\cancel{\cdot}} 2) \cdot 3 = \underline{6}$ $1 \cdot (2 \overset{6}{\cancel{\cdot}} 3) = \underline{6}$
$2 \cdot (3 \overset{6}{\cancel{\cdot 2}}) = \underline{12}$ $(2 \overset{6}{\cancel{\cdot}} 3) \cdot 2 = \underline{12}$	$3 \cdot (3 \overset{9}{\cancel{\cdot}} 3) = \underline{27}$ $(3 \overset{9}{\cancel{\cdot}} 3) \cdot 3 = \underline{27}$



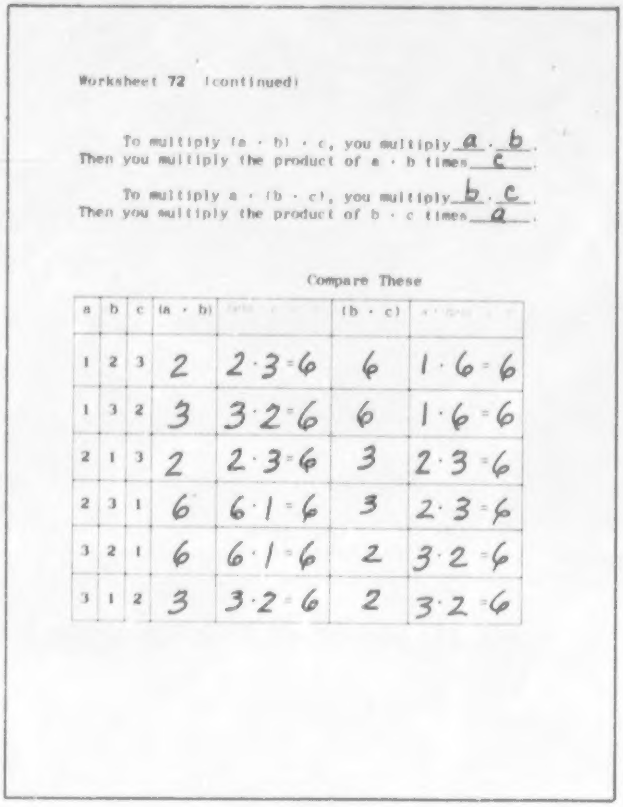
Worksheet 72
Unit 27
Name ____________

When you multiply three numbers, a, b, and c, does it make any difference how you group them?

Yes [✓ No]

Fill in this table: Compare these

a	b	c	$(a \cdot b)$	[illegible]	$(b \cdot c)$	[illegible]
2	3	5	6	$6 \cdot 5 = 30$	15	$2 \cdot 15 = 30$
2	5	3	10	$10 \cdot 3 = 30$	15	$2 \cdot 15 = 30$
3	5	2	15	$15 \cdot 2 = 30$	10	$3 \cdot 10 = 30$
3	2	5	6	$6 \cdot 5 = 30$	10	$3 \cdot 10 = 30$
5	3	2	15	$15 \cdot 2 = 30$	6	$5 \cdot 6 = 30$
5	2	3	10	$10 \cdot 3 = 30$	6	$5 \cdot 6 = 30$



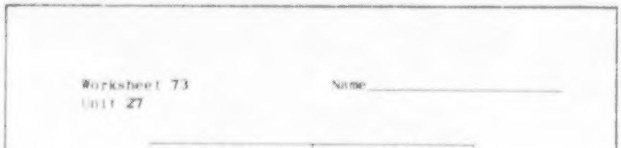
Worksheet 72 (continued)

To multiply $(a \cdot b) \cdot c$, you multiply a · b.
Then you multiply the product of $a \cdot b$ times c.

To multiply $a \cdot (b \cdot c)$, you multiply b · c.
Then you multiply the product of $b \cdot c$ times a.

Compare These

a	b	c	$(a \cdot b)$	[illegible]	$(b \cdot c)$	[illegible]
1	2	3	2	$2 \cdot 3 = 6$	6	$1 \cdot 6 = 6$
1	3	2	3	$3 \cdot 2 = 6$	6	$1 \cdot 6 = 6$
2	1	3	2	$2 \cdot 3 = 6$	3	$2 \cdot 3 = 6$
2	3	1	6	$6 \cdot 1 = 6$	3	$2 \cdot 3 = 6$
3	2	1	6	$6 \cdot 1 = 6$	2	$3 \cdot 2 = 6$
3	1	2	3	$3 \cdot 2 = 6$	2	$3 \cdot 2 = 6$



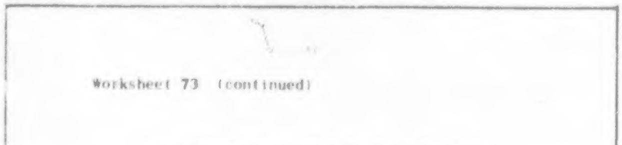
Worksheet 73
Unit 27
Name ____________

$20 = 2 \cdot 10$ $30 = 3 \cdot 10$

Fill in the blanks.

$40 = 4 \cdot 10$ $60 = 6 \cdot 10$ $80 = 8 \cdot 10$
$50 = 5 \cdot 10$ $70 = 7 \cdot 10$ $90 = 9 \cdot 10$
Bonus: $100 = 10 \cdot 10$

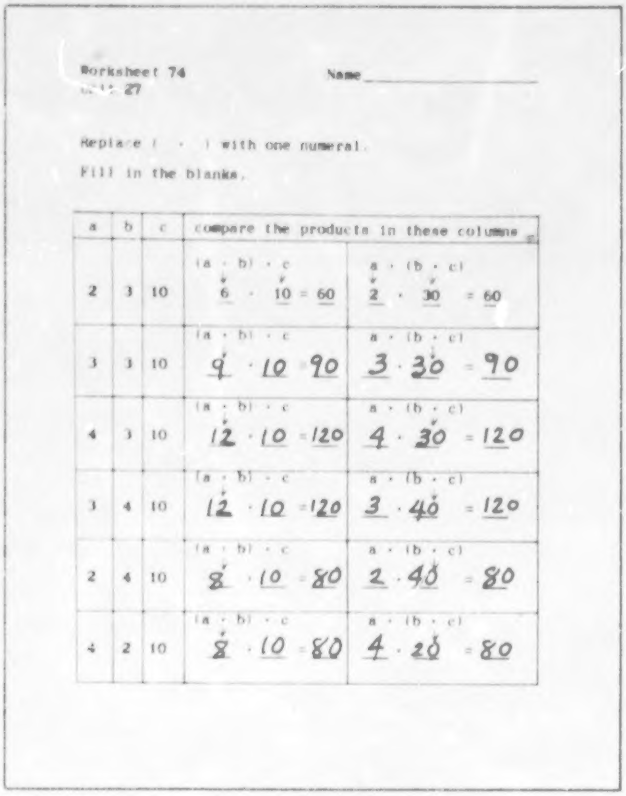
$20 = 2 \cdot 10$ so $3 \cdot 20 = 3 \cdot (2 \cdot 10)$
$30 = 3 \cdot 10$ so $4 \cdot 30 = 4 \cdot (3 \cdot 10)$
$70 = 7 \cdot 10$ so $6 \cdot 70 = 5 \cdot (7 \cdot 10)$
$50 = 5 \cdot 10$ so $9 \cdot 50 = 9 \cdot (5 \cdot 10)$
$40 = 4 \cdot 10$ so $5 \cdot 40 = 5 \cdot (4 \cdot 10)$
$70 = 7 \cdot 10$ so $5 \cdot 70 = 5 \cdot (7 \cdot 10)$
$90 = 9 \cdot 10$ so $6 \cdot 90 = 6 \cdot (9 \cdot 10)$



Worksheet 73 (continued)

$3 \cdot 2 = 6$ $3 \cdot 20 = 60$
$4 \cdot 2 = 8$ $4 \cdot 20 = 80$
$5 \cdot 2 = 10$ $5 \cdot 20 = 100$
$6 \cdot 2 = 12$ $6 \cdot 20 = 120$
$7 \cdot 2 = 14$ $7 \cdot 20 = 140$
$8 \cdot 2 = 16$ $8 \cdot 20 = 160$
$9 \cdot 2 = 18$ $9 \cdot 20 = 180$

$2 \cdot 3 = 6$ $2 \cdot 30 = 60$
$3 \cdot 3 = 9$ $3 \cdot 30 = 90$
$4 \cdot 3 = 12$ $4 \cdot 30 = 120$
$5 \cdot 3 = 15$ $5 \cdot 30 = 150$
$6 \cdot 3 = 18$ $6 \cdot 30 = 180$
$8 \cdot 3 = 24$ $8 \cdot 30 = 240$
$9 \cdot 3 = 27$ $9 \cdot 30 = 270$



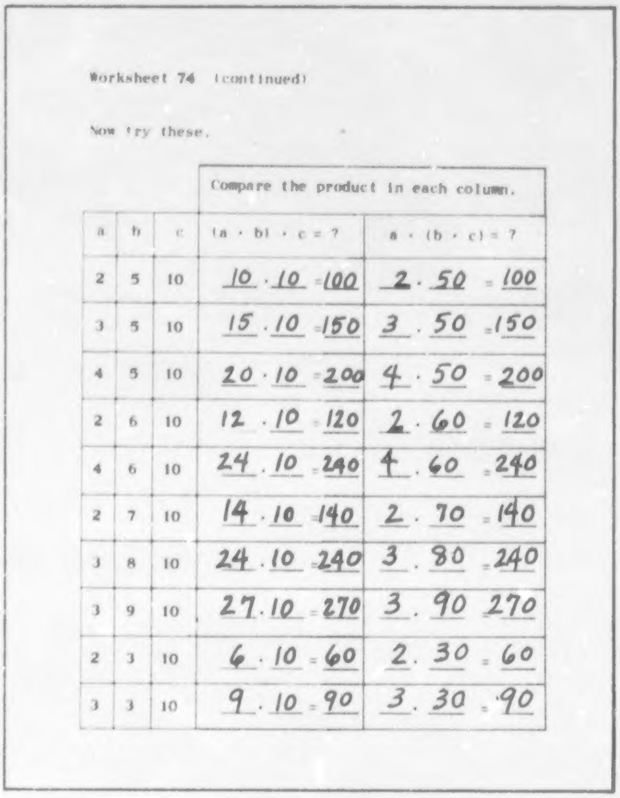
Worksheet 74 Name ____________

Unit 27

Replace (·) with one numeral.

Fill in the blanks.

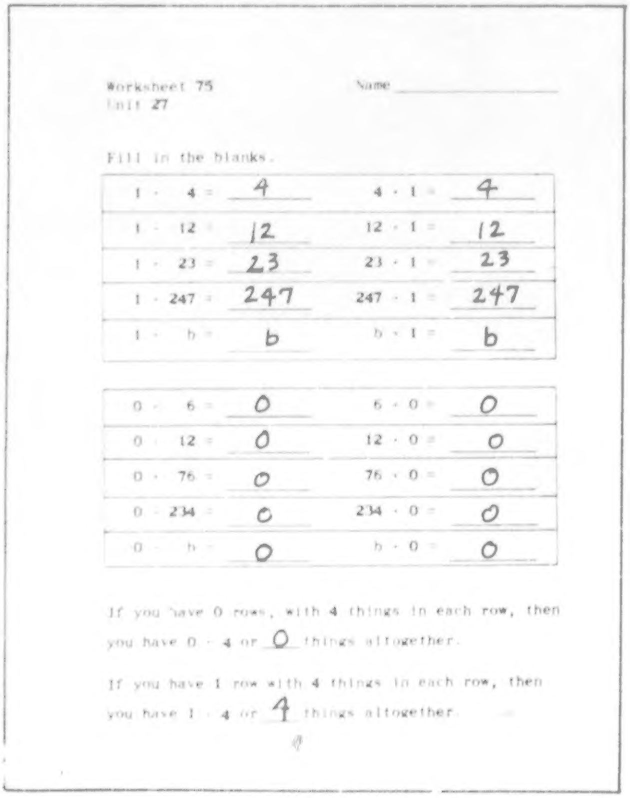
a	b	c	compare the products in these columns	
2	3	10	(a · b) · c 6 · 10 = 60	a · (b · c) 2 · 30 = 60
3	3	10	(a · b) · c 9 · 10 = 90	a · (b · c) 3 · 30 = 90
4	3	10	(a · b) · c 12 · 10 = 120	a · (b · c) 4 · 30 = 120
3	4	10	(a · b) · c 12 · 10 = 120	a · (b · c) 3 · 40 = 120
2	4	10	(a · b) · c 8 · 10 = 80	a · (b · c) 2 · 40 = 80
4	2	10	(a · b) · c 8 · 10 = 80	a · (b · c) 4 · 20 = 80



Worksheet 74 (continued)

Now try these.

			Compare the product in each column.	
a	b	c	(a · b) · c = ?	a · (b · c) = ?
2	5	10	10 · 10 = 100	2 · 50 = 100
3	5	10	15 · 10 = 150	3 · 50 = 150
4	5	10	20 · 10 = 200	4 · 50 = 200
2	6	10	12 · 10 = 120	2 · 60 = 120
4	6	10	24 · 10 = 240	4 · 60 = 240
2	7	10	14 · 10 = 140	2 · 70 = 140
3	8	10	24 · 10 = 240	3 · 80 = 240
3	9	10	27 · 10 = 270	3 · 90 270
2	3	10	6 · 10 = 60	2 · 30 = 60
3	3	10	9 · 10 = 90	3 · 30 = 90



Worksheet 75 Name ____________

Unit 27

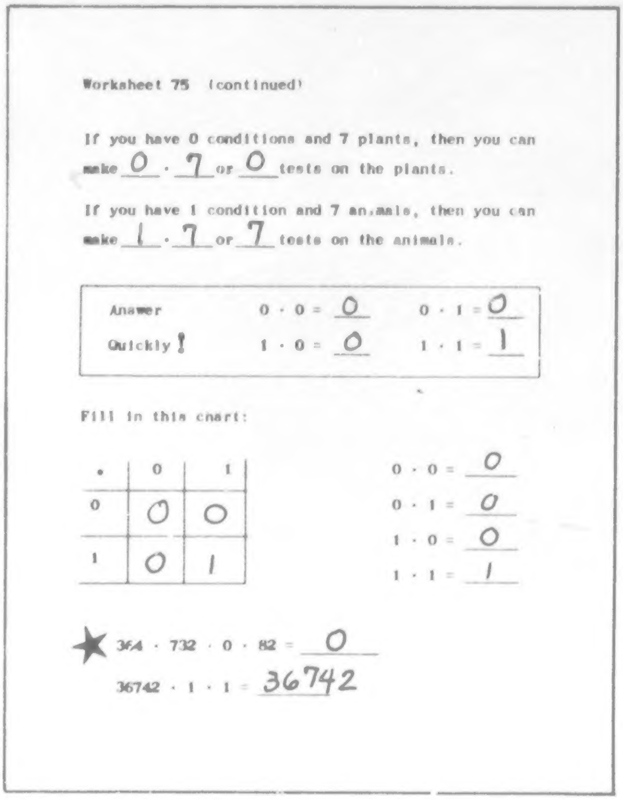
Fill in the blanks.

1 · 4 = 4	4 · 1 = 4
1 · 12 = 12	12 · 1 = 12
1 · 23 = 23	23 · 1 = 23
1 · 247 = 247	247 · 1 = 247
1 · b = b	b · 1 = b

0 · 6 = 0	6 · 0 = 0
0 · 12 = 0	12 · 0 = 0
0 · 76 = 0	76 · 0 = 0
0 · 234 = 0	234 · 0 = 0
0 · b = 0	b · 0 = 0

If you have 0 rows, with 4 things in each row, then you have 0 · 4 or 0 things altogether.

If you have 1 row with 4 things in each row, then you have 1 · 4 or 4 things altogether.



Worksheet 75 (continued)

If you have 0 conditions and 7 plants, then you can make 0 · 7 or 0 tests on the plants.

If you have 1 condition and 7 animals, then you can make 1 · 7 or 7 tests on the animals.

Answer	0 · 0 = 0	0 · 1 = 0
Quickly !	1 · 0 = 0	1 · 1 = 1

Fill in this chart:

·	0	1
0	0	0
1	0	1

0 · 0 = 0

0 · 1 = 0

1 · 0 = 0

1 · 1 = 1

★ 364 · 732 · 0 · 82 = 0

36742 · 1 · 1 = 36742

Section 3

Developing the Multiplication Vertical Algorithm

SECTION 3 DEVELOPING THE MULTIPLICATION VERTICAL ALGORITHM

PURPOSE

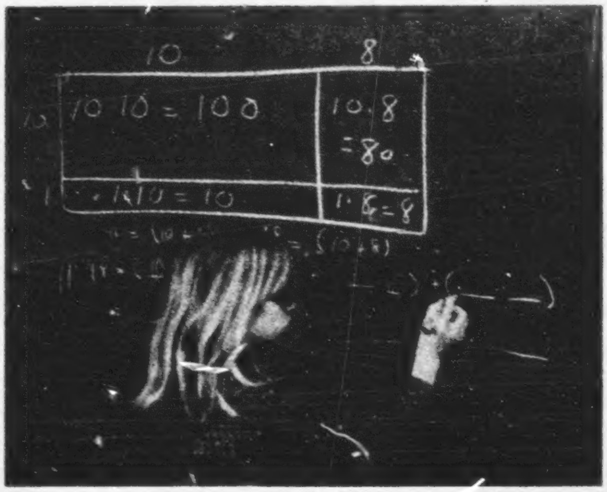
In this section the students:

- Continue working with the multiplication algorithm using arrays. They match parts of multiplication sentences to corresponding parts of arrays. They multiply two-digit factors using arrays.
- Learn to rename a two-digit factor as the sum of two smaller factors.
- Multiply one- and two-digit factors in vertical form using the multiplication algorithm.

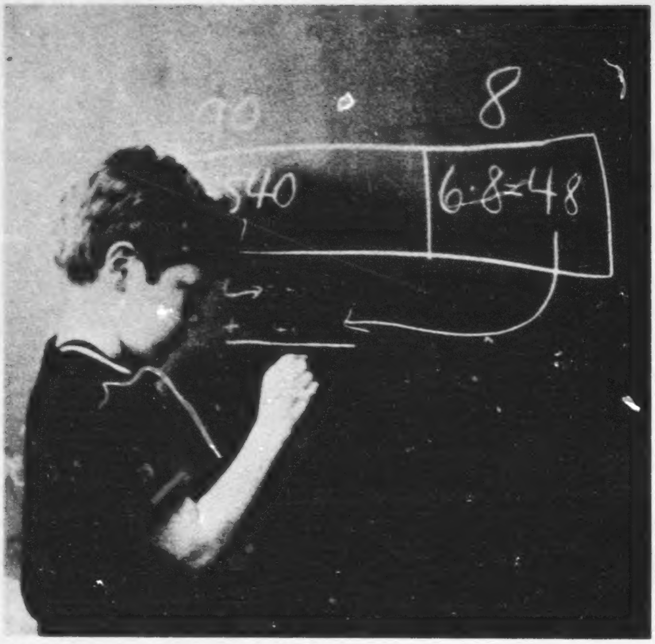
COMMENTARY

This section of the unit develops the vertical form of the multiplication algorithm. The students complete many of the worksheets independently or with partners; other work is done in class discussions.

Lesson 13 introduces an algorithm using arrays. Rectangular arrays illustrate how a two-digit factor can be renamed as the sum of two smaller factors. For example, 3 · 12 can be illustrated with an array and rewritten as 3 · (7 + 5) or 3 · (10 + 2).

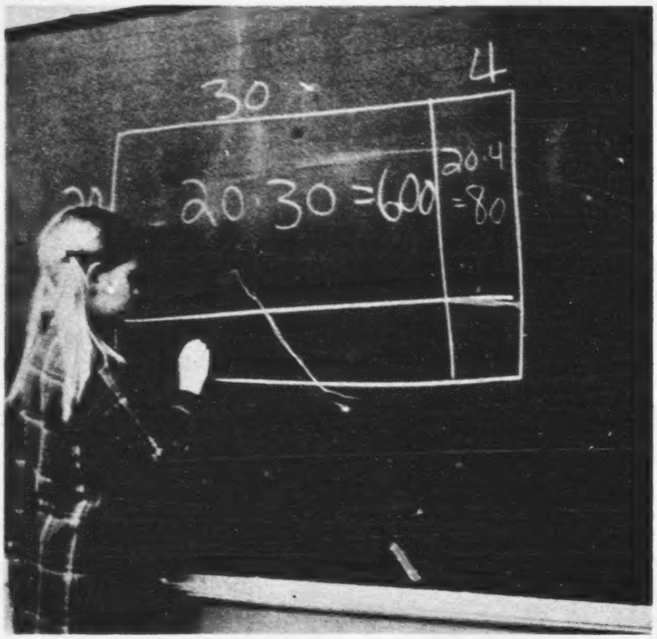


In Lesson 14 the students multiply two-digit factors using arrays. They work problems where both factors are two-digit numbers. Again they rename the two-digit factor as the sum of two smaller one-digit numbers. The students fill in missing parts of multiplication sentences that correspond to parts of the array.



Lesson 15 introduces the vertical algorithm. Arrays are used to illustrate each step of the algorithm. The students add in vertical form the subproducts of partitioned arrays to find the total product. They draw arrows to match parts of the partitioned array with parts of the problem written in vertical form. As a class discussion you introduce the procedure of multiplying without arrays.

In Lesson 16 the students use the vertical algorithm to work problems where both factors are two-digit numbers. The worksheets are completed as a class discussion with you illustrating the work at the chalkboard.

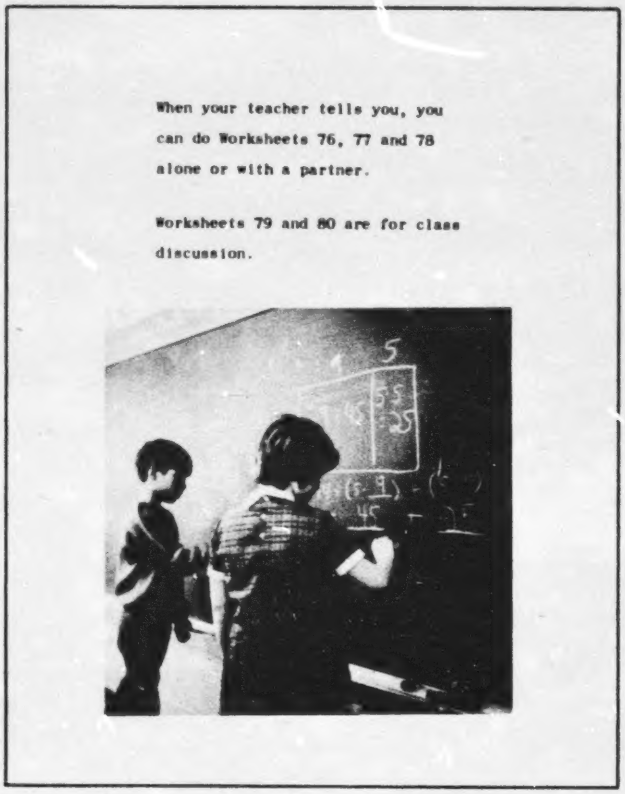


Lesson 17 provides additional practice in using the vertical multiplication algorithm. The students match parts of multiplication sentences and parts of problems written in vertical form to parts of arrays. The lesson reviews concepts introduced earlier in the unit and provides practice in multiplying with the vertical algorithm.

At this time you may want one group of students to begin the job booklet. While one group of students does the job booklet, the others can do the worksheets in this section with your assistance. (See the Notes on Teaching this Unit in the unit introduction.)

Lesson 13: INTRODUCING AN ALGORITHM USING ARRAYS

The students determine an upper estimate and a lower estimate for a range of numbers that contains the product of two factors. Rectangular arrays are used to develop an algorithm for the multiplication of whole numbers. The students rename one factor as the sum of two smaller factors.



When your teacher tells you, you can do Worksheets 76, 77 and 78 alone or with a partner.

Worksheets 79 and 80 are for class discussion.

MATERIALS

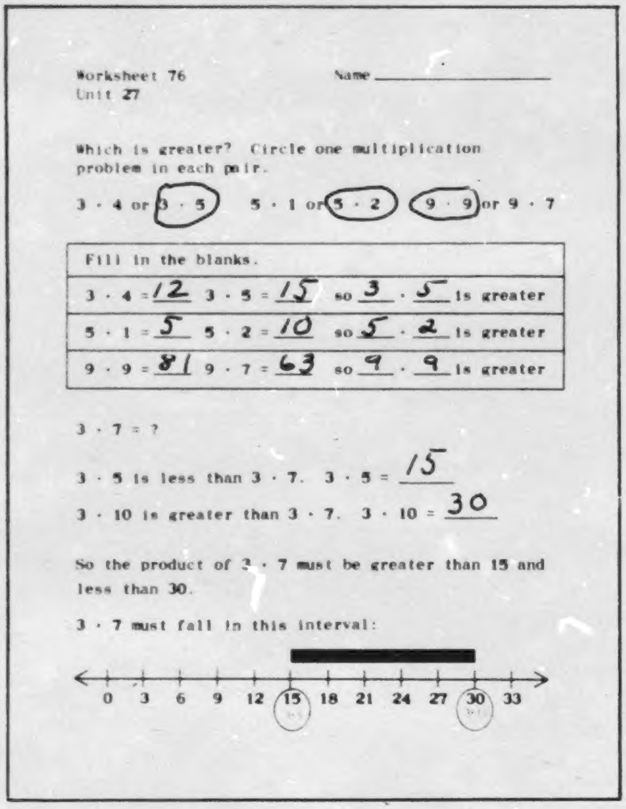
- Worksheets 76-78 (independent)
- Worksheets 79 and 80 (discussion)

PROCEDURE

Have the students complete Worksheets 76, 77 and 78 alone or with partners. When most children have completed these, discuss Worksheet 79 as a group. Then have the children complete Worksheet 80 and discuss it.

Activity A

Worksheet 76 presents the idea that, for example, 3 · 5 is greater than 3 · 4. If any students have difficulty with this idea, you may want to discuss more examples before the students continue the independent work. The students determine an upper and a lower estimate for a product of two



Worksheet 76 Name ____________
Unit 27

Which is greater? Circle one multiplication problem in each pair.

3 · 4 or [✓ 3 · 5] 5 · 1 or [✓ 5 · 2] [✓ 9 · 9] or 9 · 7

Fill in the blanks.

3 · 4 = 12 3 · 5 = 15 so 3 · 5 is greater

5 · 1 = 5 5 · 2 = 10 so 5 · 2 is greater

9 · 9 = 81 9 · 7 = 63 so 9 · 9 is greater

3 · 7 = ?

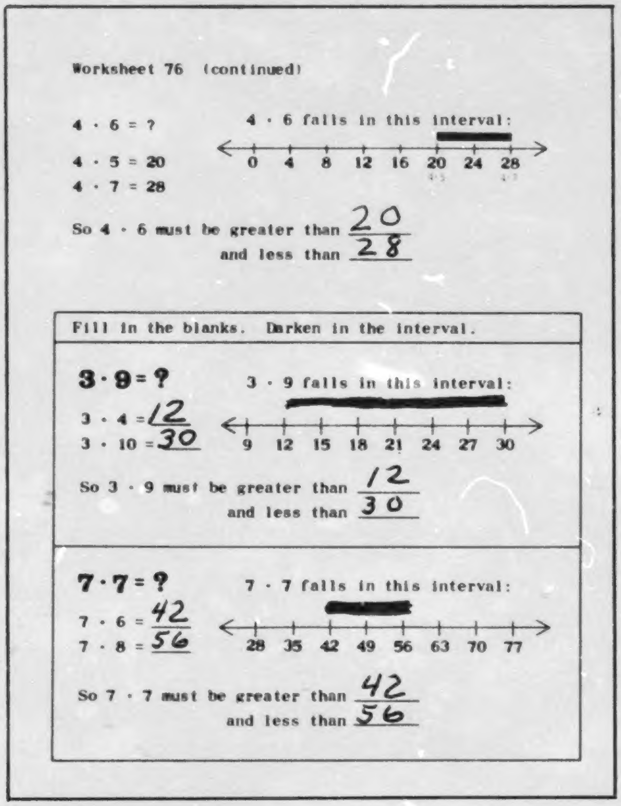
3 · 5 is less than 3 · 7. 3 · 5 = 15

3 · 10 is greater than 3 · 7. 3 · 10 = 30

So the product of 3 · 7 must be greater than 15 and less than 30.

3 · 7 must fall in this interval:

0 3 6 9 12 15 18 21 24 27 30 33



Worksheet 76 (continued)

4 · 6 = ? 4 · 6 falls in this interval:

4 · 5 = 20

4 · 7 = 28

0 4 8 12 16 20 24 28

So 4 · 6 must be greater than 20
and less than 28

Fill in the blanks. Darken in the interval.

3 · 9 = ? 3 · 9 falls in this interval:

3 · 4 = 12

3 · 10 = 30

9 12 15 18 21 24 27 30

So 3 · 9 must be greater than 12
and less than 30

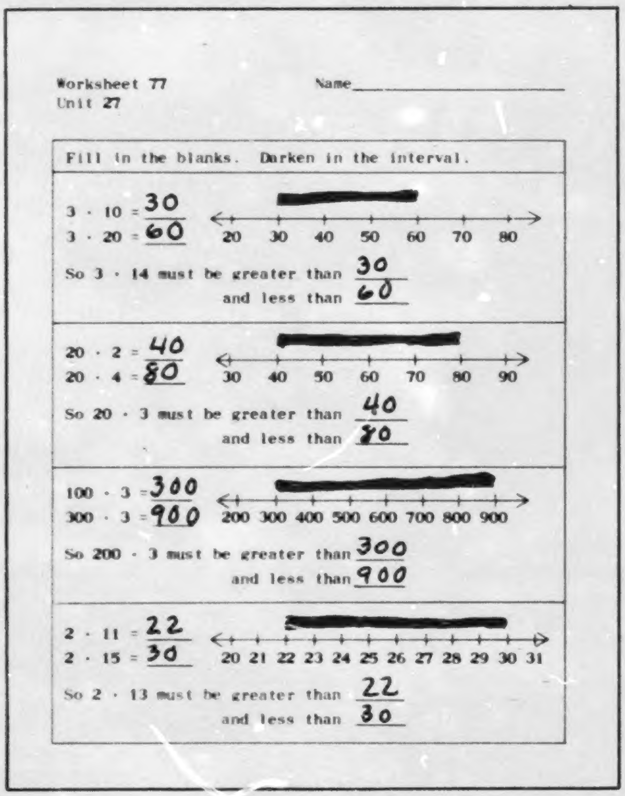
7 · 7 = ? 7 · 7 falls in this interval:

7 · 6 = 42

7 · 8 = 56

28 35 42 49 56 63 70 77

So 7 · 7 must be greater than 42
and less than 56



Worksheet 77 Name ____________
Unit 27

Fill in the blanks. Darken in the interval.

3 · 10 = 30

3 · 20 = 60

20 30 40 50 60 70 80

So 3 · 14 must be greater than 30
and less than 60

20 · 2 = 40

20 · 4 = 80

30 40 50 60 70 80 90

So 20 · 3 must be greater than 40
and less than 80

100 · 3 = 300

300 · 3 = 900

200 300 400 500 600 700 800 900

So 200 · 3 must be greater than 300
and less than 900

2 · 11 = 22

2 · 15 = 30

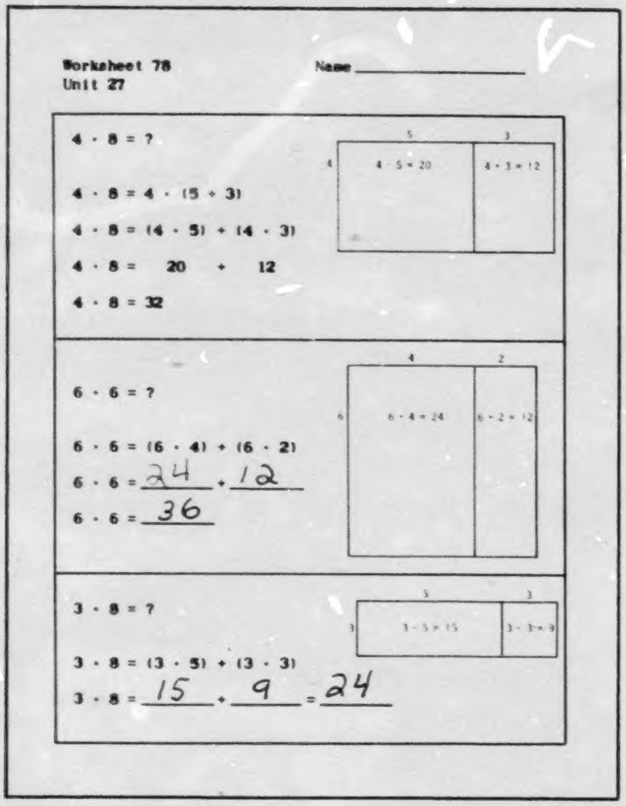
20 21 22 23 24 25 26 27 28 29 30 31

So 2 · 13 must be greater than 22
and less than 30

factors. For example, 3 · 7 is greater than 3 · 5, but is less than 3 · 10. So the product of 3 · 7 falls between the products of 3 · 5 (15) and 3 · 10 (30) on a number line. The students find the interval (range of numbers) that contains the product of two factors. (This worksheet does not raise the question of shortening that range.)

Worksheet 77 contains more problems where the students must find the range of numbers that a product falls within.

Worksheet 78 uses rectangular arrays to introduce an algorithm for multiplying whole numbers. One factor is renamed as the sum of two smaller factors. This algorithm is a helpful technique the children can use to multiply numbers greater than the basic multiplication facts. This worksheet involves numbers less than 10 where the algorithm is probably not needed, but may at times be helpful. For example, the students probably know that 4 · 8 = 32. But if they should forget the product of 4 · 8, they can use the algorithm and write the problem as (4 · 5) + (4 · 3).


5
3
4
4 · 5 = 20
4 · 3 = 12
4
2
6
6 · 4 = 24
6 · 2 = 12
5
3
3
3 · 5 = 15
3 · 3 = 9

Worksheet 78 Name ________

Unit 27

4 · 8 = ?

4 · 8 = 4 · (5 + 3)

4 · 8 = (4 · 5) + (4 · 3)

4 · 8 = 20 + 12

4 · 8 = 32

6 · 6 = ?

6 · 6 = (6 · 4) + (6 · 2)

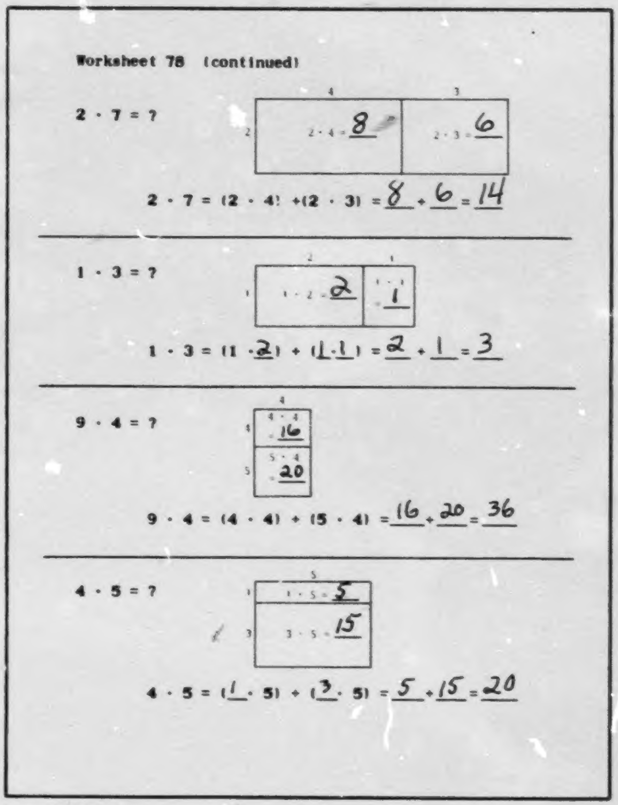
6 · 6 = 24 + 12

6 · 6 = 36

3 · 8 = ?

3 · 8 = (3 · 5) + (3 · 3)

3 · 8 = 15 + 9 = 24


4
3
2
2 · 4 = 8
2 · 3 = 6
2
1
1
1 · 2 = 2
1 · 1 = 1
4
4
4 · 4 = 16
5
5 · 4 = 20
5
1
1 · 5 = 5
3
3 · 5 = 15

Worksheet 78 (continued)

2 · 7 = ?

2 · 7 = (2 · 4) + (2 · 3) = 8 + 6 = 14

1 · 3 = ?

1 · 3 = (1 · 2) + (1 · 1) = 2 + 1 = 3

9 · 4 = ?

9 · 4 = (4 · 4) + (5 · 4) = 16 + 20 = 36

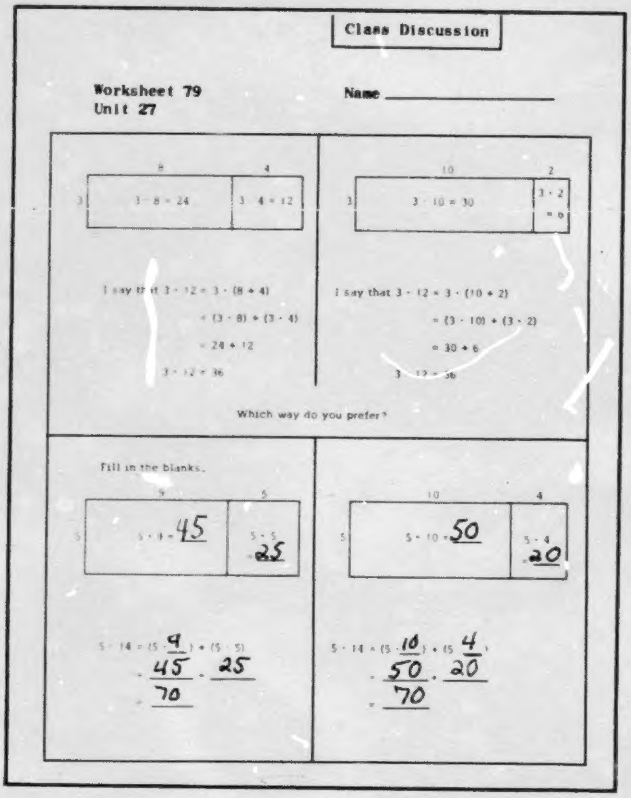
4 · 5 = ?

4 · 5 = (1 · 5) + (3 · 5) = 5 + 15 = 20

Activity B

As a group discuss the top half of Worksheet 79. This continues the multiplication algorithm developed on the previous worksheet. Two methods of partitioning a two-digit factor are shown. The factor is 12. One method is to partition it into 8 and 4. Another method is to partition it into 10 and 2. No effort is made to suggest that one way is better than the other. Have the children discuss both methods. Perhaps they might prefer to partition 12 differently — 6 and 6, for example. Encourage the students to use the method that works best for them. The students should see that partitioning a two-digit factor into smaller elements makes multiplying easier.

Have the children complete the bottom half of the worksheet. These examples show two methods of partitioning 14. Discuss both methods with the children.



Class Discussion

Worksheet 79
Unit 27

Name ____________

8
4
3
3 · 8 = 24
3 · 4 = 12

I say that 3 · 12 = 3 · (8 + 4)
= (3 · 8) + (3 · 4)
= 24 + 12
3 · 12 = 36

10
2
3
3 · 10 = 30
3 · 2
= 6

I say that 3 · 12 = 3 · (10 + 2)
= (3 · 10) + (3 · 2)
= 30 + 6
3 · 12 = 36

Which way do you prefer?

Fill in the blanks.

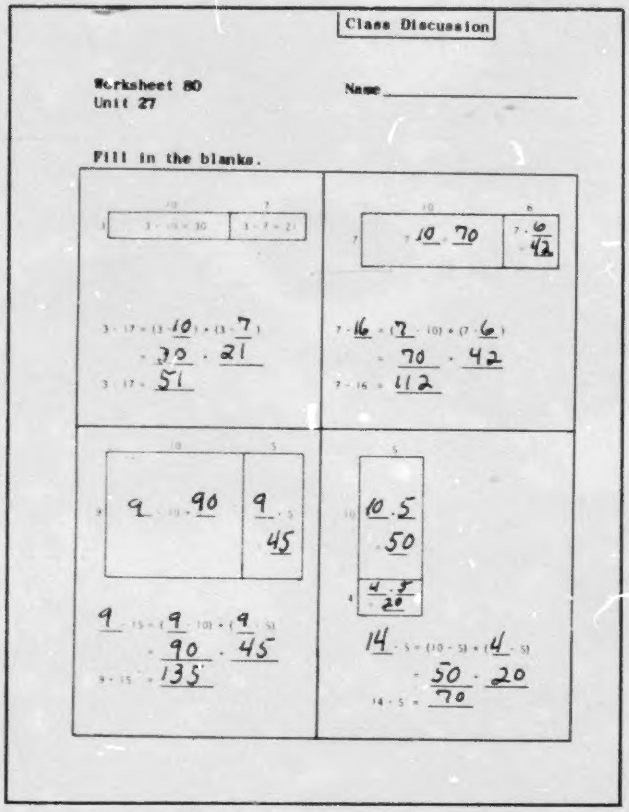
9
5
5
5 · 9 = 45
5 · 5
= 25

5 · 14 = (5 · 9) + (5 · 5)
= 45 + 25
= 70

10
4
5
5 · 10 = 50
5 · 4
= 20

5 · 14 = (5 · 10) + (5 · 4)
= 50 + 20
= 70

Give the students a few minutes to do Worksheet 80. The two-digit factors on this worksheet have been partitioned into tens and ones. When most children have completed the pages, discuss their work, explaining that it is sometimes easier to multiply a two-digit factor by partitioning it into smaller elements in a particular way, such as tens and ones.



Class Discussion

Worksheet 80
Unit 27

Name ____________

Fill in the blanks.

3 · 10 = 30
3 · 7 = 21

3 · 17 = (3 · 10) + (3 · 7)
= 30 + 21
3 · 17 = 51

7 · 10 = 70
7 · 6 = 42

7 · 16 = (7 · 10) + (7 · 6)
= 70 + 42
7 · 16 = 112

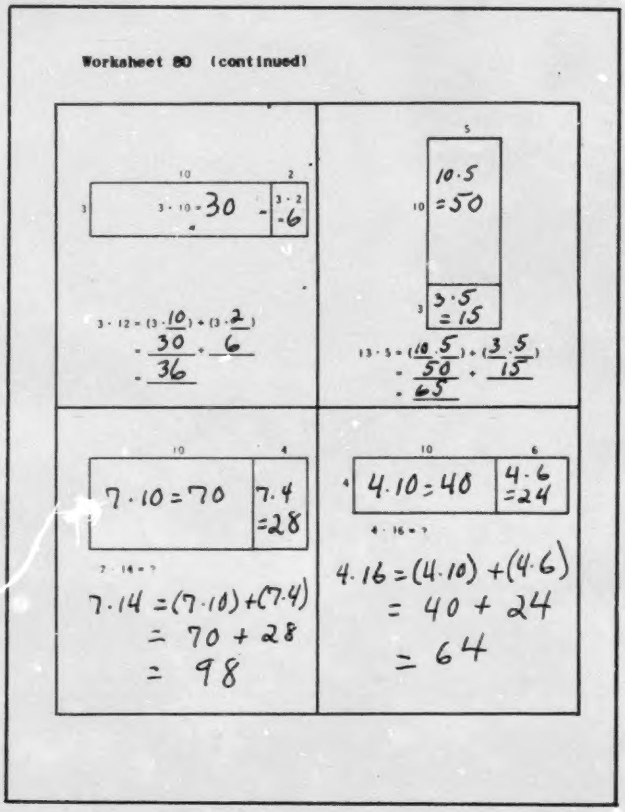
9 · 10 = 90
9 · 5 = 45

9 · 15 = (9 · 10) + (9 · 5)
= 90 + 45
9 · 15 = 135

10 · 5 = 50
4 · 5 = 20

14 · 5 = (10 · 5) + (4 · 5)
= 50 + 20
14 · 5 = 70

10
7
3
10
6
7
10
5
9
5
10
4



Worksheet 80 (continued)

3 · 10 = 30
3 · 2 = 6

3 · 12 = (3 · 10) + (3 · 2)
= 30 + 6
= 36

10 · 5 = 50
3 · 5 = 15

13 · 5 = (10 · 5) + (3 · 5)
= 50 + 15
= 65

7 · 10 = 70
7 · 4 = 28

7 · 14 = ?

7 · 14 = (7 · 10) + (7 · 4)
= 70 + 28
= 98

4 · 10 = 40
4 · 6 = 24

4 · 16 = ?

4 · 16 = (4 · 10) + (4 · 6)
= 40 + 24
= 64

10
2
3
5
10
3
10
4
10
6
4

Lesson 14: MULTIPLYING TWO-DIGIT NUMBERS USING ARRAYS

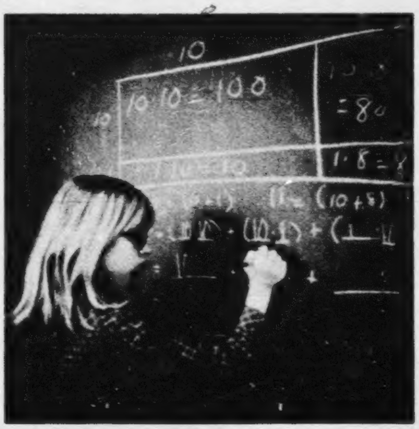
In this lesson the children complete worksheets independently. They learn to multiply two-digit numbers by partitioning arrays. In this process they rename a two-digit number as two smaller numbers.

MATERIALS

- Worksheets 81 - 84 (independent)
- Worksheet 85 (fast workers)

In this lesson all the students will do Worksheets 81, 82, 83 and 84.

Students who work fast can do Worksheet 85.

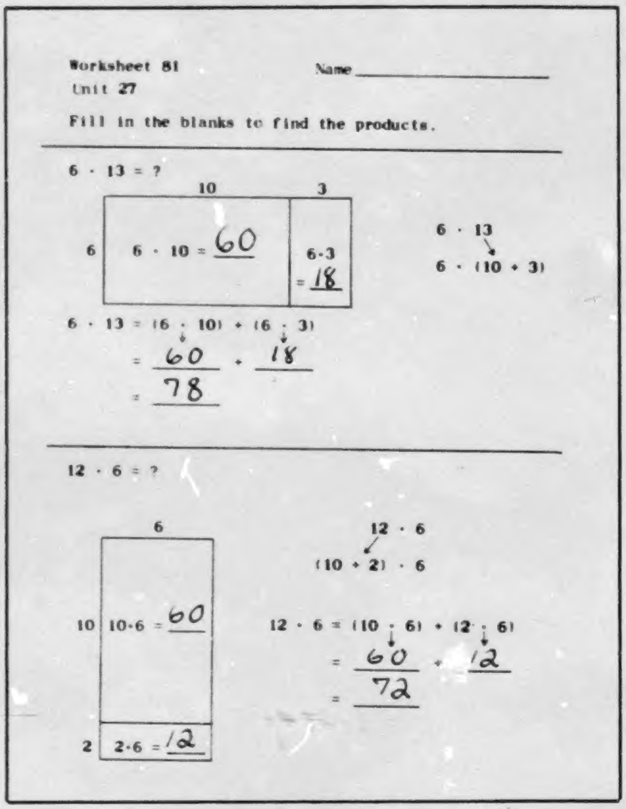


PROCEDURE

Have the children complete Worksheets 81 through 84 independently or with partners. In Activity B, you discuss two worksheets from Activity A.

Activity A

On Worksheets 81 and 82 the students do multiplication problems, each involving a two-digit factor. They use arrays that have been partitioned into smaller arrays. The two-digit factor is partitioned into tens and ones.



Worksheet 81 Name ____________________
Unit 27

Fill in the blanks to find the products.

6 · 13 = ?

	10	3
6	6 · 10 = 60	6·3 = 18

6 · 13
6 · (10 + 3)

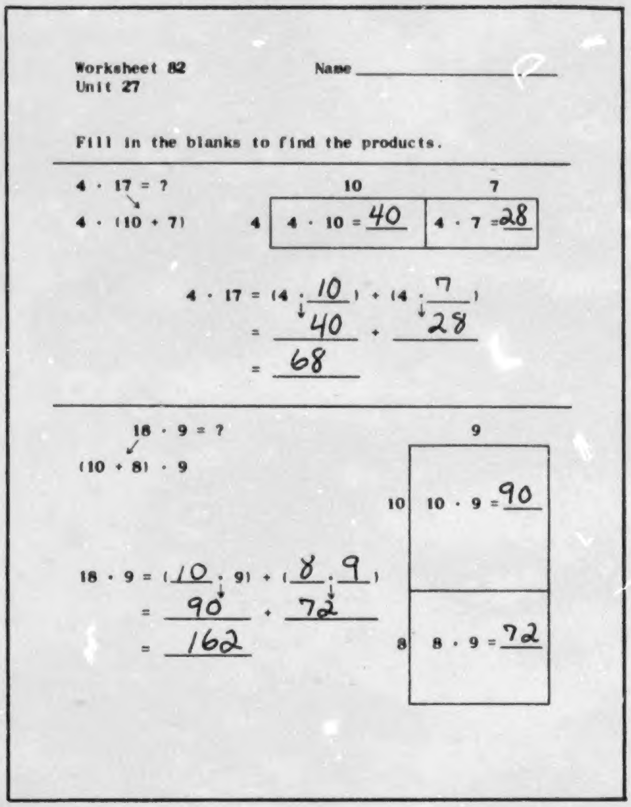
6 · 13 = (6 · 10) + (6 · 3)
= 60 + 18
= 78

12 · 6 = ?

	6
10	10·6 = 60
2	2·6 = 12

12 · 6
(10 + 2) · 6

12 · 6 = (10 · 6) + (2 · 6)
= 60 + 12
= 72



Worksheet 82 Name ____________________
Unit 27

Fill in the blanks to find the products.

4 · 17 = ?
4 · (10 + 7)

	10	7
4	4 · 10 = 40	4 · 7 = 28

4 · 17 = (4 · 10) + (4 · 7)
= 40 + 28
= 68

18 · 9 = ?
(10 + 8) · 9

	9
10	10 · 9 = 90
8	8 · 9 = 72

18 · 9 = (10 · 9) + (8 · 9)
= 90 + 72
= 162

On Worksheet 83 the students multiply problems where both factors are two-digit numbers. In each problem, a large array is partitioned into four smaller arrays of tens and ones.

On Worksheet 84 the students must partition each array themselves, and then use the arrays to find the product of each problem. In the first problem only one factor is a two-digit number. In the second problem both factors are two-digit numbers. The students can partition the arrays in any way they want. Have the students save this worksheet to discuss in Activity B.

Worksheet 83
Unit 27

Name ____________________

12 · 15 = ?

	10	5
10	10 · 10 = 100	10 · 5 = 50
2	2 · 10 = 20	2 · 5 = 10

12 · 15
↙ ↘
(10 + 2) · (10 + 5)

12 · 15 = (10 · 10) + (10 · 5) + (2 · 10) + (2 · 5)

= 100 + 50 + 20 + 10

= 180

Worksheet 83 (continued)

14 · 19 = ?

	10	9
10	10 · 10 · 100	10 · 9 · 90
4	4 · 10 · 40	4 · 9 · 36

14 · 19
↙ ↘
(10 + 4) · (10 + 9)

Fill in the multiplication sentences in the array. Then finish this problem.

14 · 19 = (10 · 10) + (10 · 9) + (4 · 10) + (4 · 9)

= 100 + 90 + 40 + 36

= 266

Worksheet 84
Unit 27

Name ____________________

Partition this array into smaller arrays. Fill in all the blanks. Find the product of 3 · 13.

3 · 13 = ?

	13	
	10	3
3	3 · 10 = 30	3 · 3 = 9

3 · (10 + 3)

3 · 13 = (3 · 10) + (3 · 3)

= 30 + 9

= 39

There are several other ways to partition the array.

Worksheet 84 (continued)

Partition this array into smaller arrays. Fill in all the blanks. Find the product of 11 · 18.

11 · 18 = ?

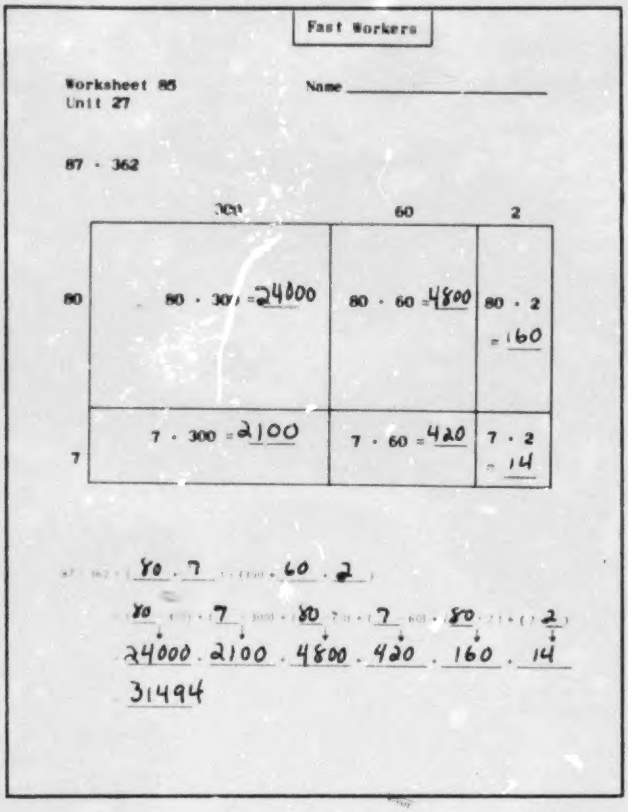
		18	
		10	8
11	10	10 · 10 = 100	10 · 8 = 80
	1	1 · 10 = 10	1 · 8 = 8

11 = (10 + 1) 18 = (10 + 8)

11 · 18 = (10 · 10) + (10 · 8) + (1 · 10) + (1 · 8)

= 100 + 80 + 10 + 8

= 198



Fast Workers

Worksheet 85
Unit 27

Name ______________________

87 · 362

	300	60	2
80	80 · 300 = 24000	80 · 60 = 4800	80 · 2 = 160
7	7 · 300 = 2100	7 · 60 = 420	7 · 2 = 14

87 · 362 = (80 + 7) · (300 + 60 + 2)

= (80 · 300) + (7 · 300) + (80 · 60) + (7 · 60) + (80 · 2) + (7 · 2)

= 24000 + 2100 + 4800 + 420 + 160 + 14

= 31494

Worksheet 85 is for fast workers. The problem on this page involves multiplying a two-digit number by a three-digit number. It may help to tell those children who try this worksheet that 80 · 300, for example, can be found my multiplying 8 · 3 and writing three zeros. Have all children save this worksheet whether or not they have completed it.

Activity B

For additional practice, you may want to spend a second class period on this lesson to discuss Worksheets 84 and 85. Have several students draw their solutions to Worksheet 84 on the chalkboard.

Draw the array from Worksheet 85 on the chalkboard. Not all students will have done this worksheet, but encourage all to help fill in the blanks to the problem. The students should see that when a multiplication problem is broken down into small parts, it is not as difficult as it first appears.

Lesson 15: INTRODUCING THE VERTICAL ALGORITHM

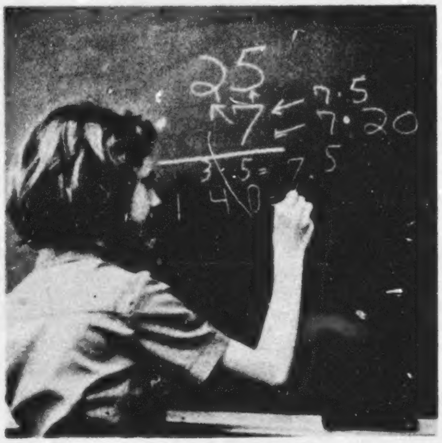
As a class the students begin writing the vertical form of a multiplication algorithm. Arrays are used to illustrate each step of the algorithm. Worksheets provide the material for the discussion. You may need to spend two class periods on this lesson.

MATERIALS

- Worksheets 86 - 90 (class discussion)

In this lesson all the worksheets will be done together as a class.

The students will do Worksheets 86, 87, 88, 89 and 90.



PROCEDURE

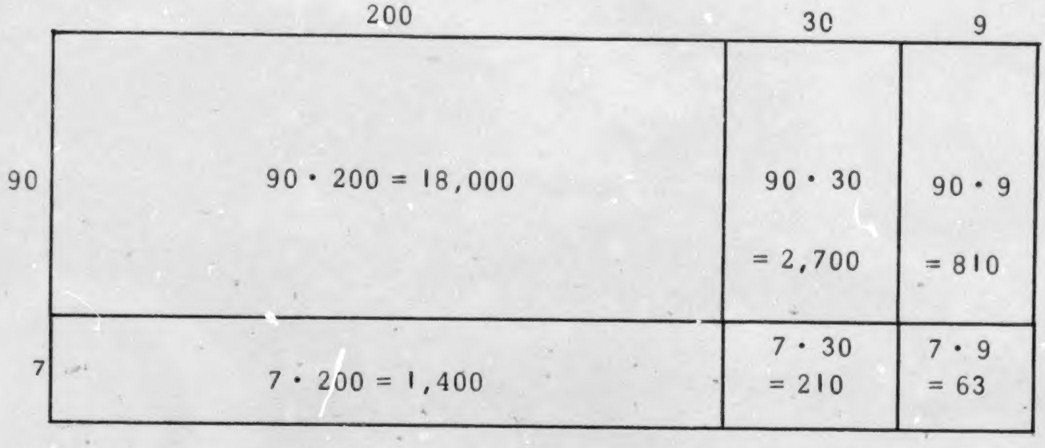
Worksheets 86 through 90 should be discussed as a group. As the students proceed through the worksheets, illustrate and explain the work at the chalkboard.

The students first work with one- and two-digit factors, but can work with larger numbers when they understand the procedure. On the next page is an example of multiplying large numbers by using an array and then the vertical algorithm.

This example shows how to multiply large numbers using an array and the vertical algorithm. The students use the same procedure with one- and two-digit factors.

① Write a multiplication problem using large numbers: 97 · 239.

② Draw an array to illustrate the problem. Partition the array:



③ Write the vertical algorithm that solves the problem.

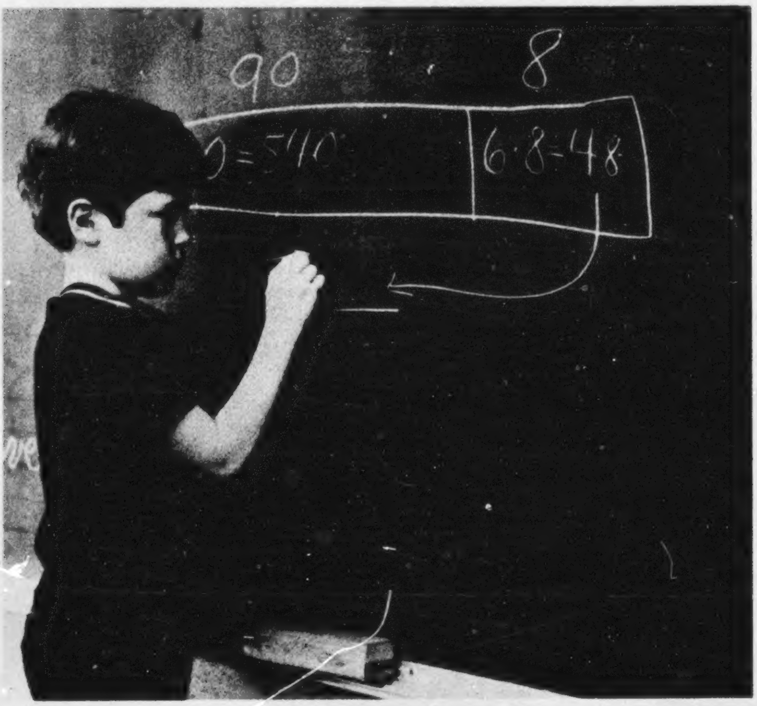
Add.

18,000 = 90 · 200
2,700 = 90 · 30
810 = 90 · 9
1,400 = 7 · 200
210 = 7 · 30
63 = 7 · 9

23,183 = 97 · 239

Worksheet 86 reviews the procedure of adding subproducts of partitioned arrays to find the total product. These subproducts are totaled in vertical form outside of the array.

On Worksheet 87 the students draw arrows showing the association between part of an array and part of a multiplication algorithm. The students also fill in missing parts of multiplication sentences. The vertical form of the algorithm is shown in each problem, but nothing is said about it to the students at this stage.



Class Discussion

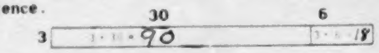
Worksheet 86
Unit 27

Name ____________

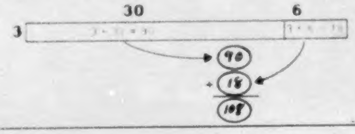
① Here is an array for 3 · 36.

It is partitioned into tens and ones.

Fill in the product for each multiplication sentence.



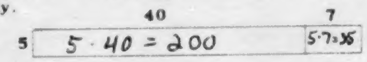
② Add the two products to get the total product.



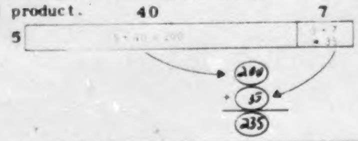
③ Here is an array for 5 · 47.

It is partitioned into tens and ones.

Write a multiplication sentence in each smaller array.



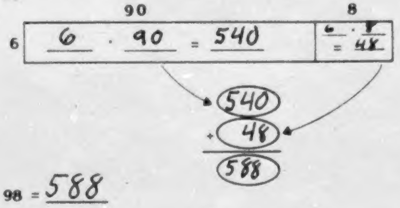
④ Add the product of each smaller array to get the total product.



Worksheet 86 (continued)

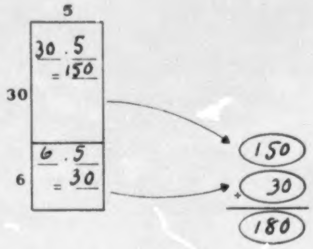
⑥ Do these problems.

6 · 98



6 · 98 = 588

36 · 5



36 · 5 = 180

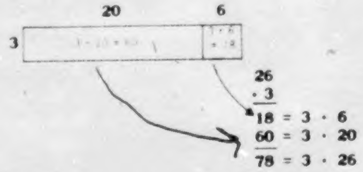
Class Discussion

Worksheet 87
Unit 27

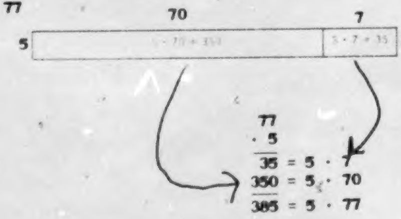
Name ____________

Draw arrows from the part of the array that matches the part of the multiplication record.

3 · 26



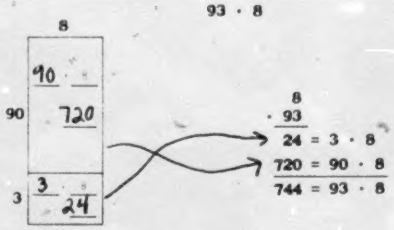
5 · 77



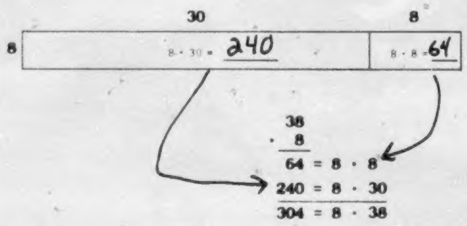
Worksheet 87 (continued)

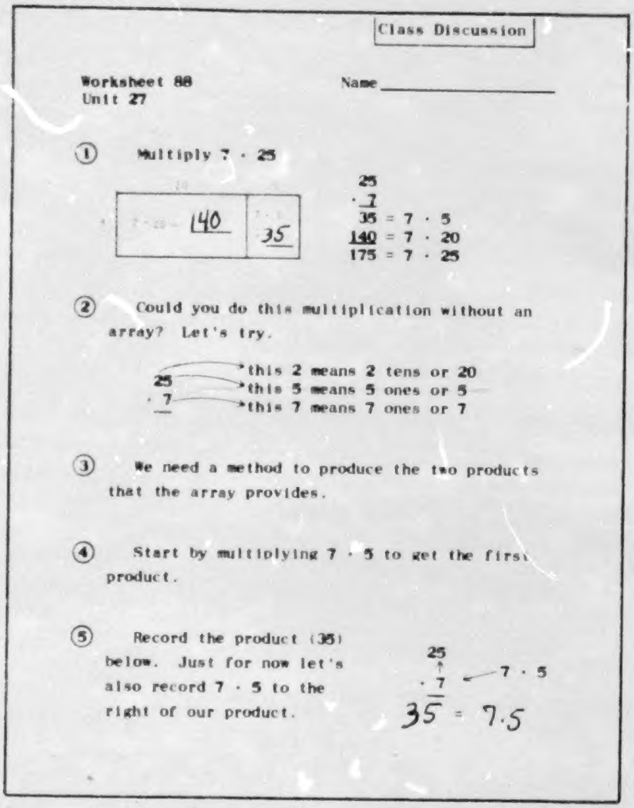
Draw arrows. Also fill in the blanks.

93 · 8



8 · 38





Class Discussion

Worksheet 88
Unit 27

Name____________________

① Multiply 7 · 25

140	35

$$\begin{array}{r} 25 \\ \cdot\ \underline{7} \\ 35 = 7 \cdot 5 \\ \underline{140} = 7 \cdot 20 \\ 175 = 7 \cdot 25 \end{array}$$

② Could you do this multiplication without an array? Let's try.

$$\begin{array}{r} 25 \\ \cdot\ \underline{7} \end{array}$$

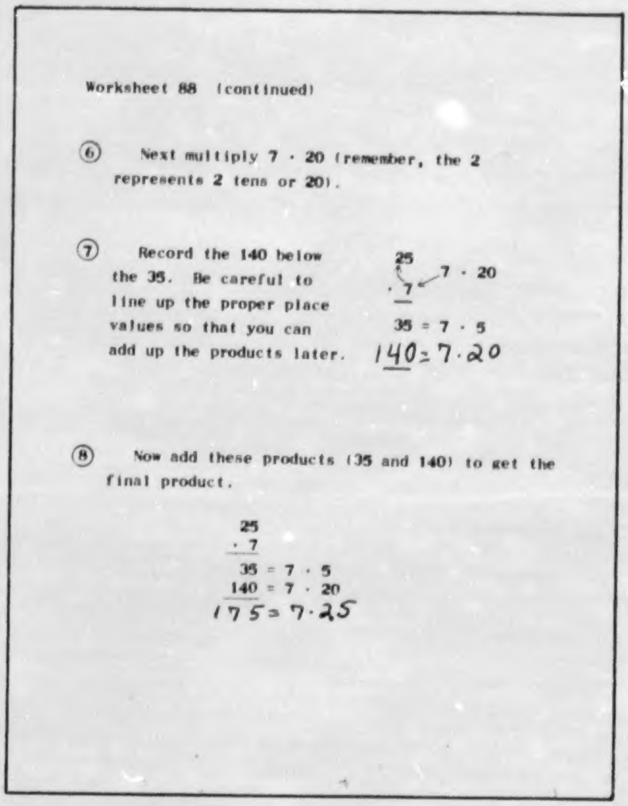
→ this 2 means 2 tens or 20
→ this 5 means 5 ones or 5
→ this 7 means 7 ones or 7

③ We need a method to produce the two products that the array provides.

④ Start by multiplying 7 · 5 to get the first product.

⑤ Record the product (35) below. Just for now let's also record 7 · 5 to the right of our product.

$$\begin{array}{r} 25 \\ \uparrow \\ \cdot\ \underline{7} \leftarrow 7 \cdot 5 \\ 35 = 7 \cdot 5 \end{array}$$



Worksheet 88 (continued)

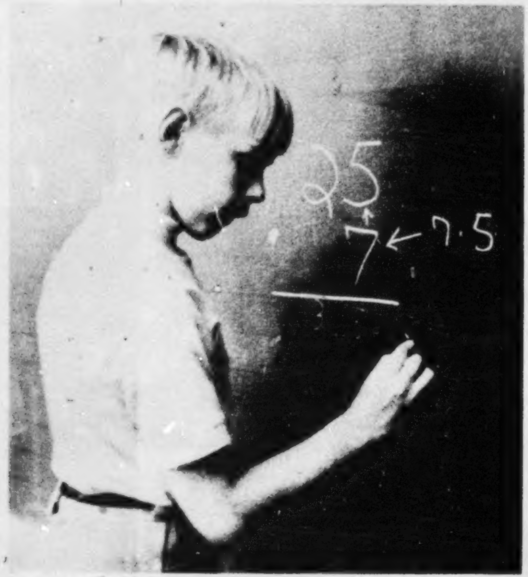
⑥ Next multiply 7 · 20 (remember, the 2 represents 2 tens or 20).

⑦ Record the 140 below the 35. Be careful to line up the proper place values so that you can add up the products later.

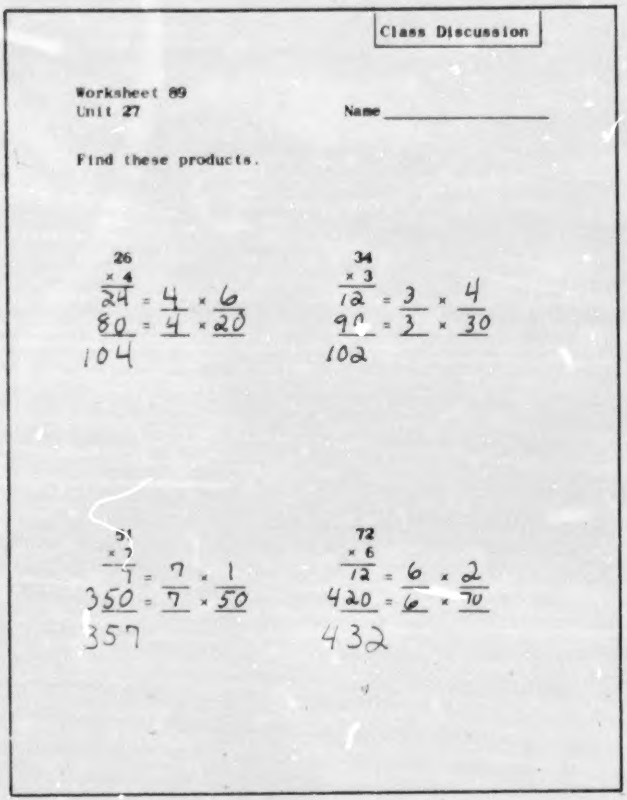
$$\begin{array}{r} 25 \\ \cdot\ \underline{7} \leftarrow 7 \cdot 20 \\ 35 = 7 \cdot 5 \\ \underline{140} = 7 \cdot 20 \end{array}$$

⑧ Now add these products (35 and 140) to get the final product.

$$\begin{array}{r} 25 \\ \underline{\cdot\ 7} \\ 35 = 7 \cdot 5 \\ \underline{140} = 7 \cdot 20 \\ 175 = 7 \cdot 25 \end{array}$$



Worksheet 88 should be worked and discussed with the whole class. This worksheet moves from the array directly to the multiplication algorithm. Be sure to emphasize the meaning of each digit and where each partial product comes from. You may want to refer back to the array often, to show how our mechanical means of multiplying corresponds to the array.



Class Discussion

Worksheet 89
Unit 27

Name ____________________

Find these products.

26
× 4
24 = 4 × 6
80 = 4 × 20
104

34
× 3
12 = 3 × 4
90 = 3 × 30
102

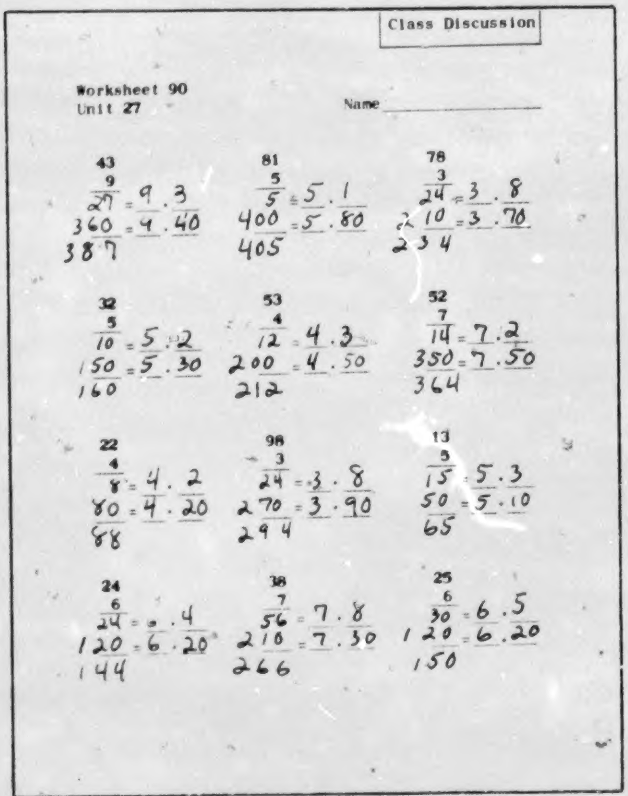
51
× 7
7 = 7 × 1
350 = 7 × 50
357

72
× 6
12 = 6 × 2
420 = 6 × 70
432

Worksheet 89 provides practice in using the algorithm. You may want to help the students get started with the first problem.

Speed and skill in using the multiplication algorithm come only through much continued practice. If you decide that great skill in using this algorithm is important for your children, then you will need to provide daily practice of a problem or two throughout the remainder of the year.

Worksheet 90 provides additional practice in multiplying a two-digit factor by a one-digit factor using the vertical algorithm.



Class Discussion

Worksheet 90
Unit 27

Name ____________________

43
9
27 = 9 . 3
360 = 4 . 40
387

81
5
5 = 5 . 1
400 = 5 . 80
405

78
3
24 = 3 . 8
2 10 = 3 . 70
234

32
5
10 = 5 . 2
150 = 5 . 30
160

53
4
12 = 4 . 3
200 = 4 . 50
212

52
7
14 = 7 . 2
350 = 7 . 50
364

22
4
8 = 4 . 2
80 = 4 . 20
88

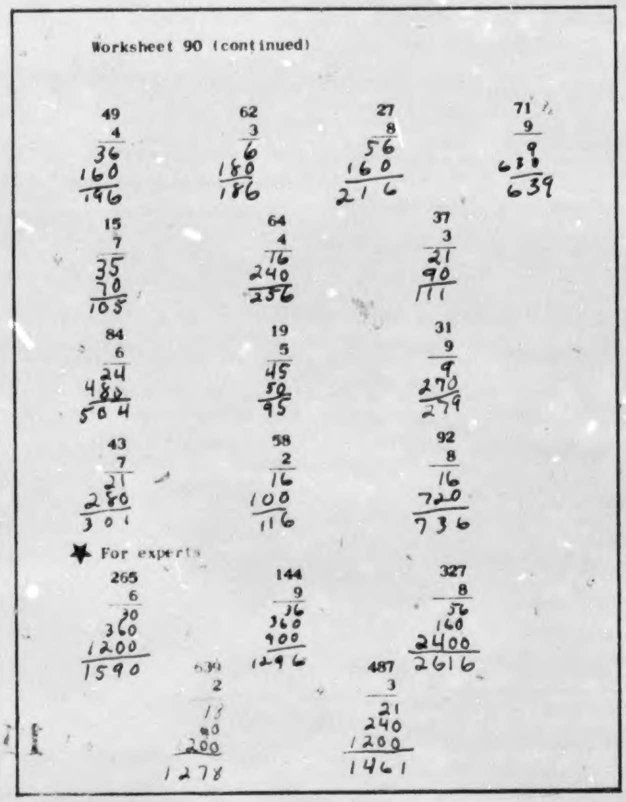
98
3
24 = 3 . 8
270 = 3 . 90
294

13
5
15 = 5 . 3
50 = 5 . 10
65

24
6
24 = . 4
120 = 6 . 20
144

38
7
56 = 7 . 8
210 = 7 . 30
266

25
6
30 = 6 . 5
120 = 6 . 20
150



Worksheet 90 (continued)

49
4
36
160
196

62
3
6
180
186

27
8
56
160
216

71
9
9
630
639

15
7
35
70
105

64
4
16
240
256

37
3
21
90
111

84
6
24
480
504

19
5
45
50
95

31
9
9
270
279

43
7
21
280
301

58
2
16
100
116

92
8
16
720
736

★ For experts

265
6
30
360
1200
1590

144
9
36
360
900
1296

327
8
56
160
2400
2616

639
2
18
60
1200
1278

487
3
21
240
1200
1461

Lesson 16: THE MULTIPLICATION ALGORITHM WHEN BOTH FACTORS ARE TWO-DIGIT NUMBERS

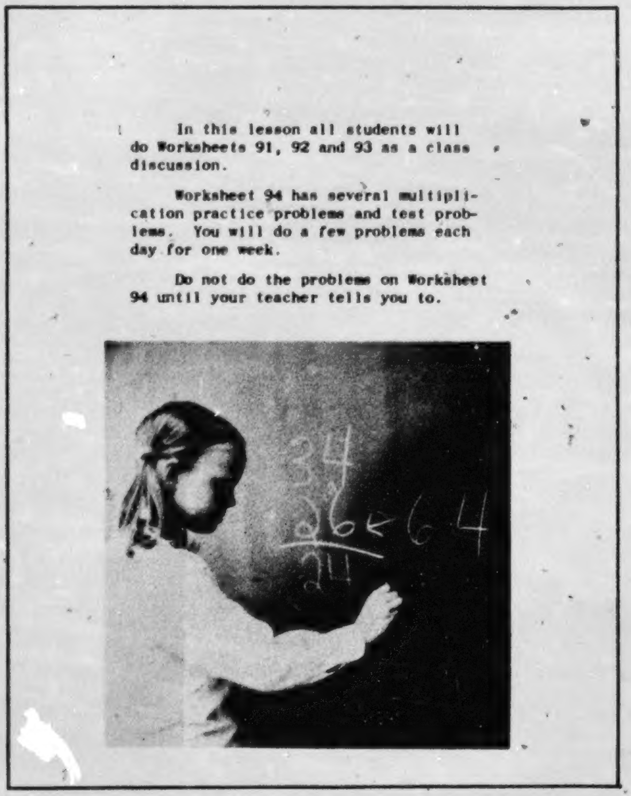
As a class the students continue studying the vertical algorithm. They work problems where both factors are two-digit numbers.

MATERIALS

- Worksheets 91-93 (class discussion)
- Worksheet 94 (independent practice problems)

PROCEDURE

Worksheets 91, 92 and 93 should be completed as a group. Illustrate the work on the chalkboard as the students read and fill in their worksheets. Allow them time to work part of a problem before the answer is given. Worksheet 94 involves practice problems and test problems similar to what the children did in Unit 24. The students should do the practice problems each day before class time. Then give them a few minutes during class to work the test problems. The answers should be posted so the children can check their work.

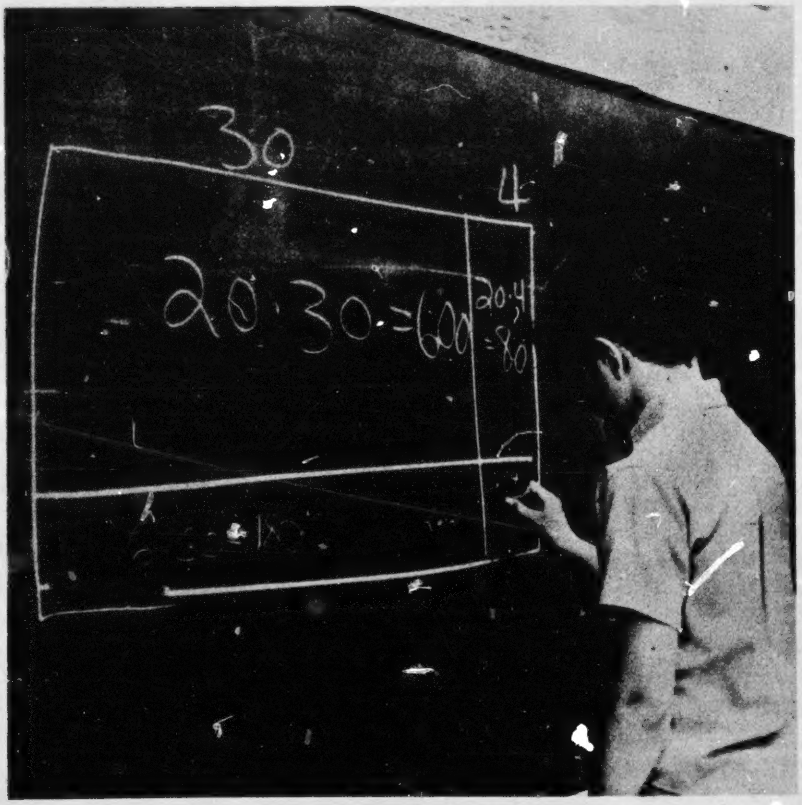


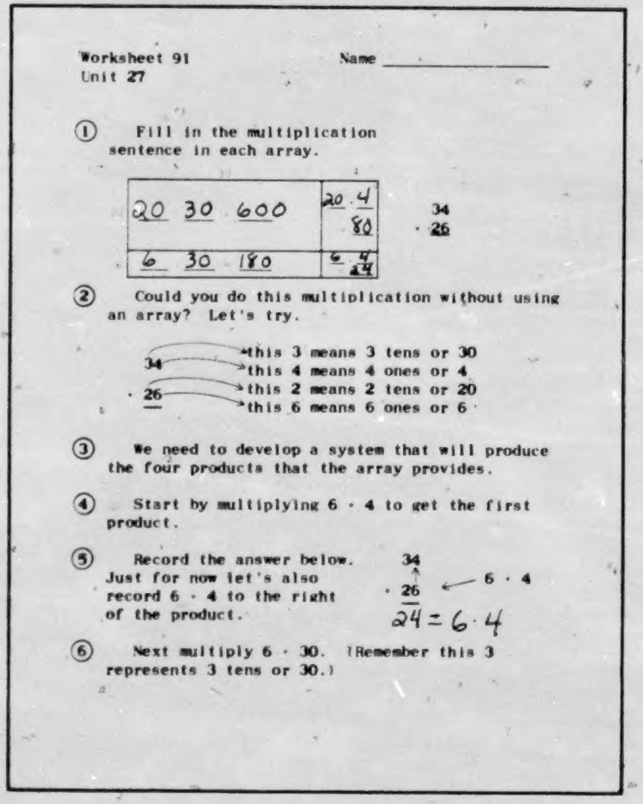
In this lesson all students will do Worksheets 91, 92 and 93 as a class discussion.

Worksheet 94 has several multiplication practice problems and test problems. You will do a few problems each day for one week.

Do not do the problems on Worksheet 94 until your teacher tells you to.

Worksheet 91 introduces the students to the procedure of multiplying two 2-digit factors. Go through the worksheet with the students, allowing them time to complete each step before the answer is given.





Worksheet 91 Name ____________
Unit 27

① Fill in the multiplication sentence in each array.

20 30 600	20 · 4 80
6 30 180	6 · 4 24

34
· 26

② Could you do this multiplication without using an array? Let's try.

34 → this 3 means 3 tens or 30
→ this 4 means 4 ones or 4
· 26 → this 2 means 2 tens or 20
→ this 6 means 6 ones or 6

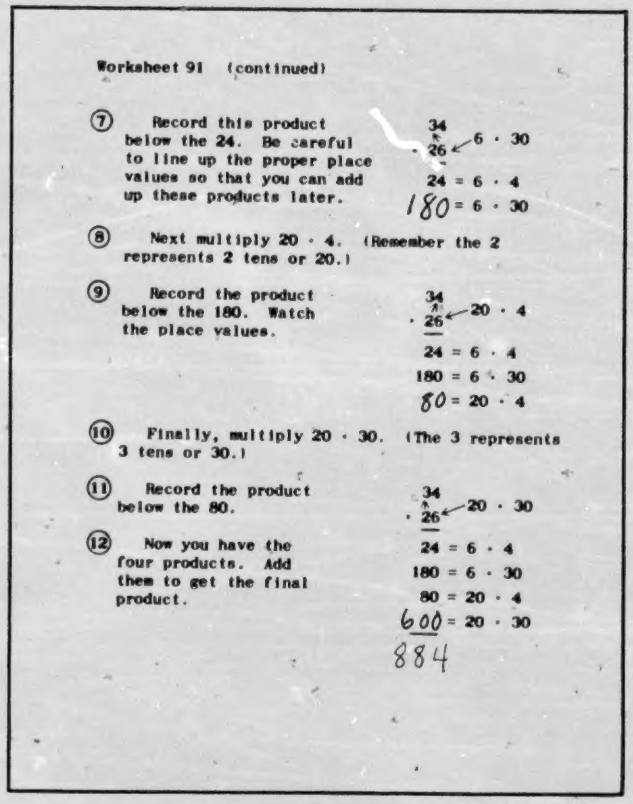
③ We need to develop a system that will produce the four products that the array provides.

④ Start by multiplying 6 · 4 to get the first product.

⑤ Record the answer below. Just for now let's also record 6 · 4 to the right of the product.

34
· 26 ← 6 · 4
24 = 6 · 4

⑥ Next multiply 6 · 30. (Remember this 3 represents 3 tens or 30.)



Worksheet 91 (continued)

⑦ Record this product below the 24. Be careful to line up the proper place values so that you can add up these products later.

34
· 26 ← 6 · 30
24 = 6 · 4
180 = 6 · 30

⑧ Next multiply 20 · 4. (Remember the 2 represents 2 tens or 20.)

⑨ Record the product below the 180. Watch the place values.

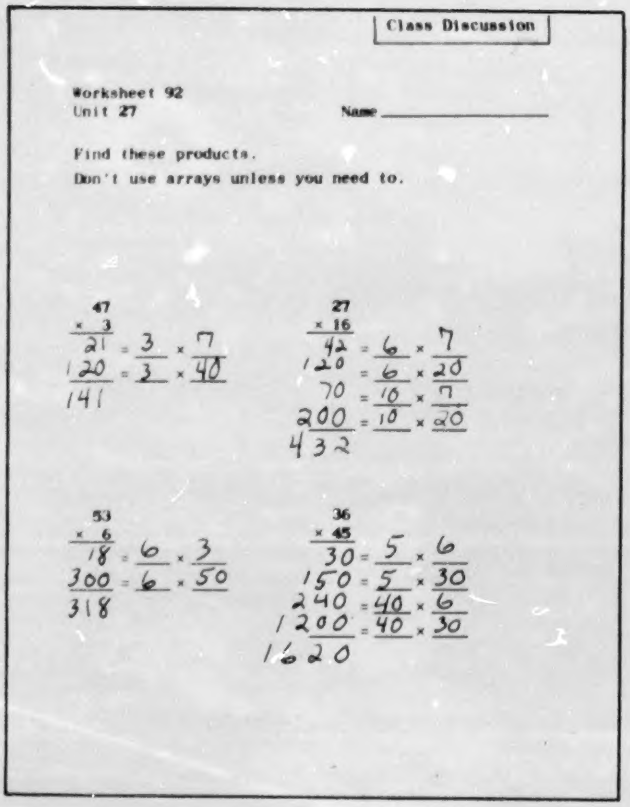
34
· 26 ← 20 · 4
24 = 6 · 4
180 = 6 · 30
80 = 20 · 4

⑩ Finally, multiply 20 · 30. (The 3 represents 3 tens or 30.)

⑪ Record the product below the 80.

⑫ Now you have the four products. Add them to get the final product.

34
· 26 ← 20 · 30
24 = 6 · 4
180 = 6 · 30
80 = 20 · 4
600 = 20 · 30
884



Class Discussion

Worksheet 92
Unit 27
Name ____________

Find these products.
Don't use arrays unless you need to.

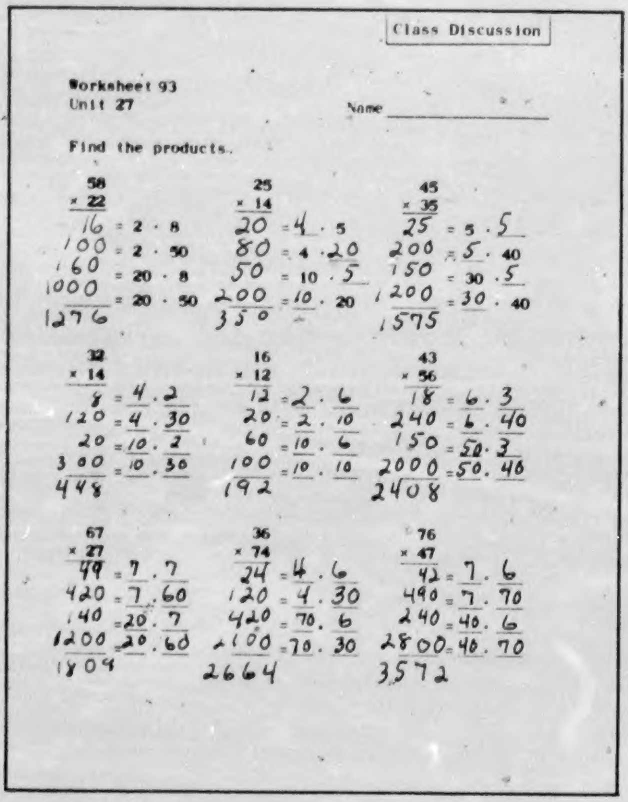
47
× 3
21 = 3 × 7
120 = 3 × 40
141

27
× 16
42 = 6 × 7
120 = 6 × 20
70 = 10 × 7
200 = 10 × 20
432

53
× 6
18 = 6 × 3
300 = 6 × 50
318

36
× 45
30 = 5 × 6
150 = 5 × 30
240 = 40 × 6
1200 = 40 × 30
1620

Worksheet 92 provides practice with multiplication in the vertical form. Let the students complete one problem at a time; after each is completed, let students explain the steps.



Class Discussion

Worksheet 93
Unit 27
Name ____________

Find the products.

58
× 22
16 = 2 · 8
100 = 2 · 50
160 = 20 · 8
1000 = 20 · 50
1276

25
× 14
20 = 4 · 5
80 = 4 · 20
50 = 10 · 5
200 = 10 · 20
350

45
× 35
25 = 5 · 5
200 = 5 · 40
150 = 30 · 5
1200 = 30 · 40
1575

32
× 14
8 = 4 · 2
120 = 4 · 30
20 = 10 · 2
300 = 10 · 30
448

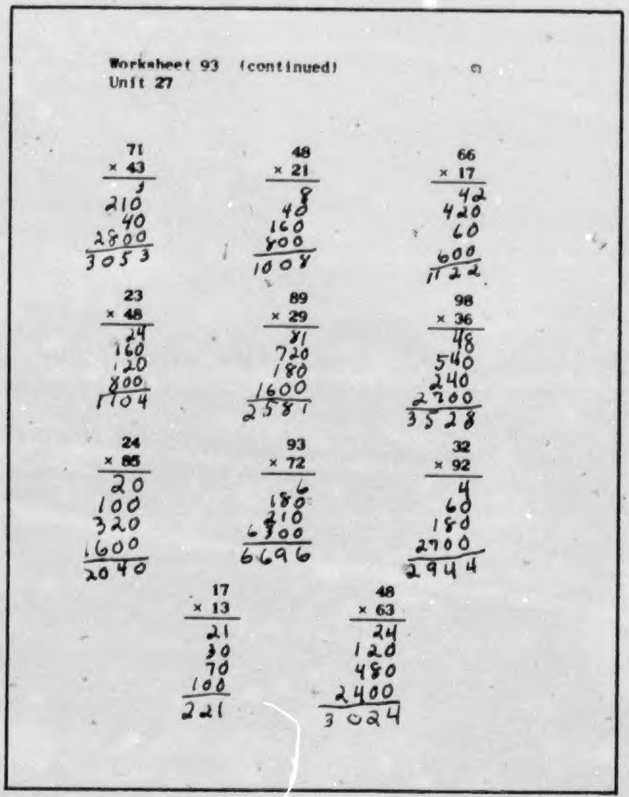
16
× 12
12 = 2 · 6
20 = 2 · 10
60 = 10 · 6
100 = 10 · 10
192

43
× 56
18 = 6 · 3
240 = 6 · 40
150 = 50 · 3
2000 = 50 · 40
2408

67
× 27
49 = 7 · 7
420 = 7 · 60
140 = 20 · 7
1200 = 20 · 60
1809

36
× 74
24 = 4 · 6
120 = 4 · 30
420 = 70 · 6
2100 = 70 · 30
2664

76
× 47
42 = 7 · 6
490 = 7 · 70
240 = 40 · 6
2800 = 40 · 70
3572



Worksheet 93 (continued)
Unit 27

71
× 43
3
210
40
2800
3053

48
× 21
8
40
160
800
1008

66
× 17
42
420
60
600
1122

23
× 48
24
160
120
800
1104

89
× 29
81
720
180
1600
2581

98
× 36
48
540
240
2700
3528

24
× 85
20
100
320
1600
2040

93
× 72
6
180
210
6300
6696

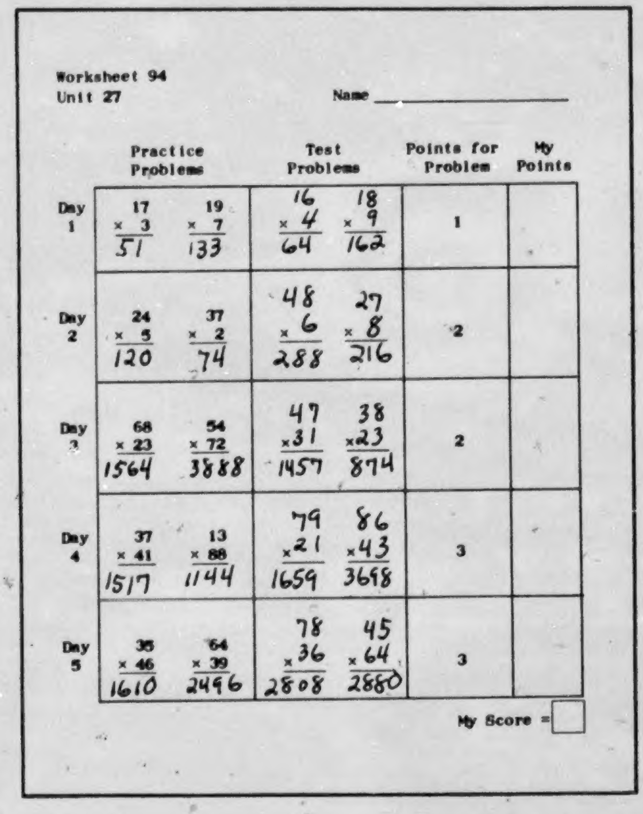
32
× 92
4
60
180
2700
2944

17
× 13
21
30
70
100
221

48
× 63
24
120
480
2400
3024

Worksheet 93 is two pages of multiplication problems where both factors are two-digit numbers. Give the students time to complete the problems, and then discuss all problems or just those the students had trouble with.

Worksheet 94 should be used for five days. Each day before class, the students complete both practice problems. Post the answers so they can check their work before doing the test problems. Then give the students a few minutes to complete both test problems. Put the answers on the chalkboard so the children can correct their work and keep track of their points. You may want to make up similar worksheets so the children can have daily practice in multiplication using the vertical algorithm. The children can use scratch paper to work the practice and test problems.



Worksheet 94
Unit 27

Name ____________

	Practice Problems		Test Problems		Points for Problem	My Points
Day 1	17 × 3 51	19 × 7 133	16 × 4 64	18 × 9 162	1	
Day 2	24 × 5 120	37 × 2 74	48 × 6 288	27 × 8 216	2	
Day 3	68 × 23 1564	54 × 72 3888	47 × 31 1457	38 × 23 874	2	
Day 4	37 × 41 1517	13 × 88 1144	79 × 21 1659	86 × 43 3698	3	
Day 5	35 × 46 1610	64 × 39 2496	78 × 36 2808	45 × 64 2880	3	

My Score = ☐

Lesson 17: MORE WORK WITH A VERTICAL ALGORITHM

On independent worksheets the students match parts of multiplication sentences and parts of problems written in vertical form with parts of arrays. Several concepts introduced earlier in the unit are reviewed. Worksheet problems provide more practice in multiplying with the vertical algorithm.

MATERIALS

- Worksheets 95-97 (independent)
- Worksheet 98 (fast workers)
- sheets of paper

PROCEDURE

In Activity A the students complete Worksheets 95, 96 and 97 independently or with partners. Fast workers can do Worksheet 98. In Activity B the students make up their own multiplication problems using the vertical algorithm they have been studying.

Activity A

On Worksheet 95 the students match parts of arrays with parts of multiplication sentences. The students fill in missing numbers in the horizontal form of each problem. This worksheet emphasizes the horizontal and vertical forms for recording a multiplication problem.

On Worksheet 96 the students also match parts of arrays with parts of vertical-form multiplication problems.

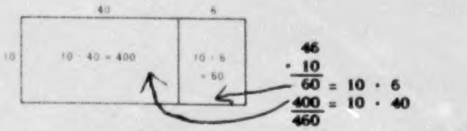
Worksheet 95
Unit 27

Name________________________

In this lesson you will do Worksheets 95, 96 and 97 by yourself or with a partner. Fast workers can also do Worksheet 98.

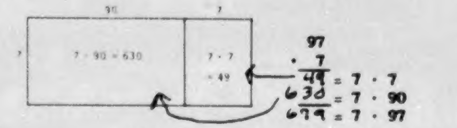
Match the parts of the array to the parts of the multiplication sentence.

Fill in the blanks.



46
· 10
60 = 10 · 6
400 = 10 · 40
460

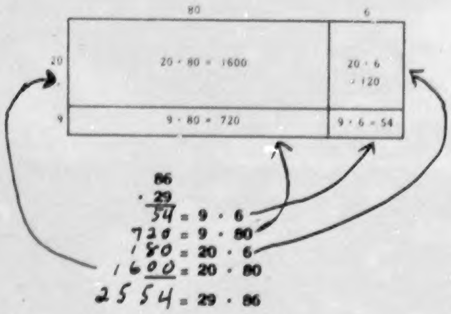
10 · 46 = (10 · 40) + (10 · 6) = 400 + 60 = 460



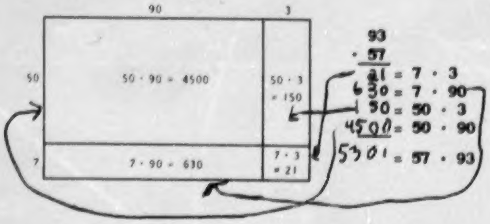
97
· 7
49 = 7 · 7
630 = 7 · 90
679 = 7 · 97

7 · 97 = (7 · 90) + (7 · 7) = 630 + 49 = 679

Worksheet 95 (continued)



86
· 29
54 = 9 · 6
720 = 9 · 80
180 = 20 · 6
1600 = 20 · 80
2554 = 29 · 86

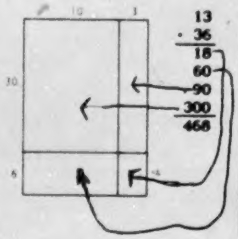


93
· 57
21 = 7 · 3
630 = 7 · 90
150 = 50 · 3
4500 = 50 · 90
5301 = 57 · 93

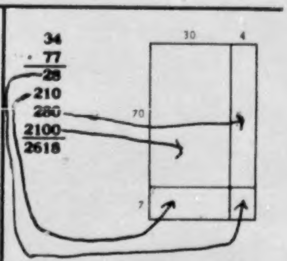
Worksheet 96
Unit 27

Name________________________

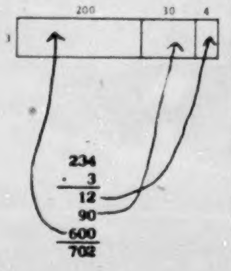
Match the parts of the array with the parts of the multiplication record.



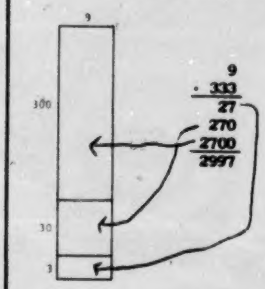
13
· 36
18
60
90
300
468



34
· 77
28
210
280
2100
2618



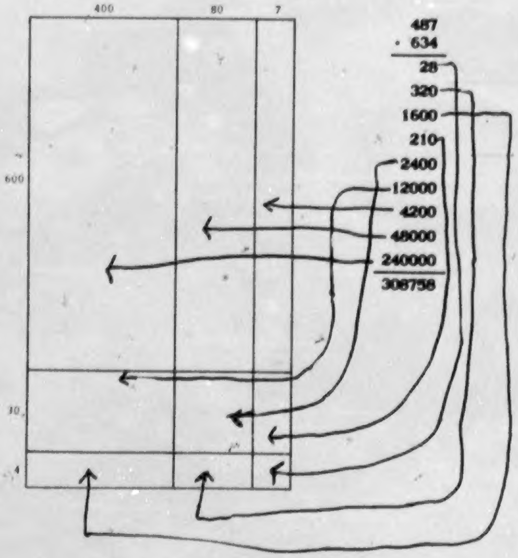
234
· 3
12
90
600
702



9
· 333
27
270
2700
2997

Worksheet 96 (continued)

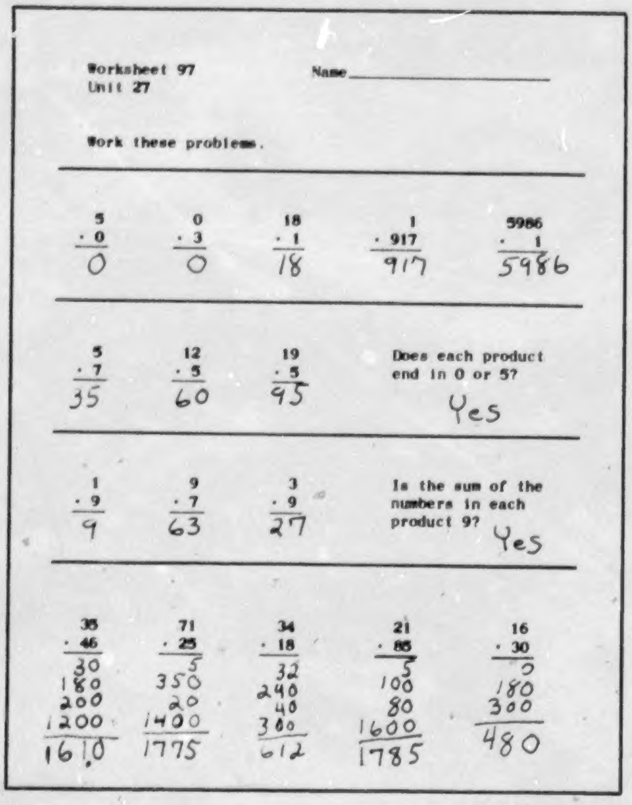
Bonus!



487
· 634
28
320
1600
210
2400
12000
4200
48000
240000
308758

Worksheet 97 provides a review of several concepts presented in this unit. These include: multiplying by 0, 1, 5 and 9, and multiplying two-digit factors in vertical form.

Worksheet 98 is for fast workers. It introduces the students to multiplication by a factor that is not a whole number. Arrays are used to illustrate the procedure.



Worksheet 97
Unit 27

Name________________

Work these problems.

$$\begin{array}{rrrrr} 5 & 0 & 18 & 1 & 5986 \\ \cdot\ 0 & \cdot\ 3 & \cdot\ 1 & \cdot\ 917 & \cdot\ \ \ 1 \\ \hline 0 & 0 & 18 & 917 & 5986 \end{array}$$

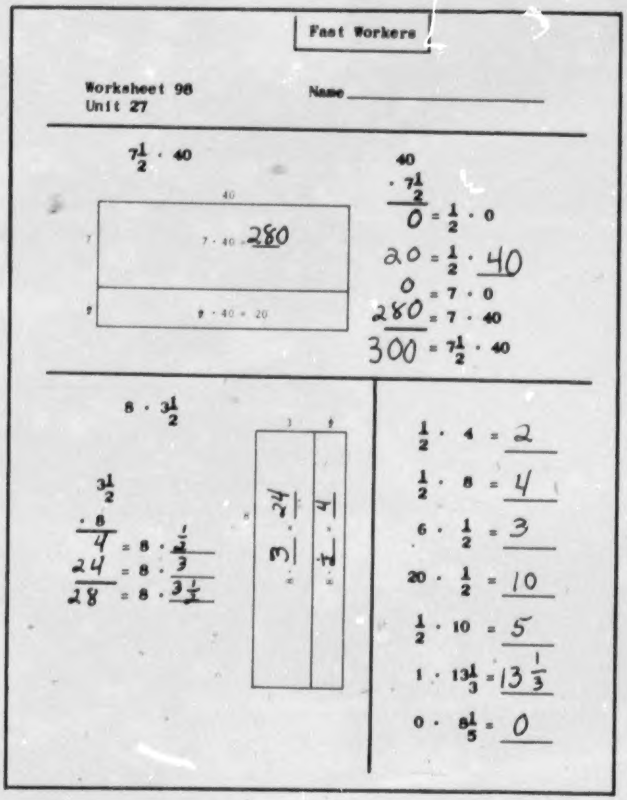
$$\begin{array}{rrr} 5 & 12 & 19 \\ \cdot\ 7 & \cdot\ 5 & \cdot\ 5 \\ \hline 35 & 60 & 95 \end{array}$$

Does each product end in 0 or 5? Yes

$$\begin{array}{rrr} 1 & 9 & 3 \\ \cdot\ 9 & \cdot\ 7 & \cdot\ 9 \\ \hline 9 & 63 & 27 \end{array}$$

Is the sum of the numbers in each product 9? Yes

$$\begin{array}{rrrrr} 35 & 71 & 34 & 21 & 16 \\ \cdot\ 46 & \cdot\ 25 & \cdot\ 18 & \cdot\ 85 & \cdot\ 30 \\ \hline 30 & 5 & 32 & 5 & 0 \\ 180 & 350 & 240 & 100 & 180 \\ 200 & 20 & 40 & 80 & 300 \\ 1200 & 1400 & 300 & 1600 & \\ \hline 1610 & 1775 & 612 & 1785 & 480 \end{array}$$



Fast Workers

Worksheet 98
Unit 27

Name________________

$7\frac{1}{2} \cdot 40$

40
7
7 · 40 = 280
½
½ · 40 = 20

$$\begin{array}{rl} 40 & \\ \cdot\ 7\frac{1}{2} & \\ \hline 0 & = \frac{1}{2} \cdot 0 \\ 20 & = \frac{1}{2} \cdot 40 \\ 0 & = 7 \cdot 0 \\ 280 & = 7 \cdot 40 \\ \hline 300 & = 7\frac{1}{2} \cdot 40 \end{array}$$

$8 \cdot 3\frac{1}{2}$

$$\begin{array}{rl} 3\frac{1}{2} & \\ \cdot\ 8 & \\ \hline 4 & = 8 \cdot \frac{1}{2} \\ 24 & = 8 \cdot 3 \\ \hline 28 & = 8 \cdot 3\frac{1}{2} \end{array}$$

3
½
8
3 · 24
½ · 4

$\frac{1}{2} \cdot 4 = $ 2

$\frac{1}{2} \cdot 8 = $ 4

$6 \cdot \frac{1}{2} = $ 3

$20 \cdot \frac{1}{2} = $ 10

$\frac{1}{2} \cdot 10 = $ 5

$1 \cdot 13\frac{1}{3} = 13\frac{1}{3}$

$0 \cdot 8\frac{1}{5} = $ 0

Activity B

When the students have completed the independent worksheets, have them find partners and create their own multiplication problems on blank paper. Encourage them to multiply numbers as large as they can using the vertical algorithm. You may want to have each child put a couple of his problems on the chalkboard for the other children to try.

Section 4

Cartesian Products

SECTION 4 CARTESIAN PRODUCTS

PURPOSE

In this section the students:

- Continue the work with Cartesian products that was introduced in Unit 25. Story-problems and worksheets provide situations where finding the Cartesian product is desirable.
- Review rectangular arrays and multiplication sentences. The students use multiplication sentences to find the Cartesian products of disjoint sets. They partition larger arrays into several smaller arrays.

COMMENTARY

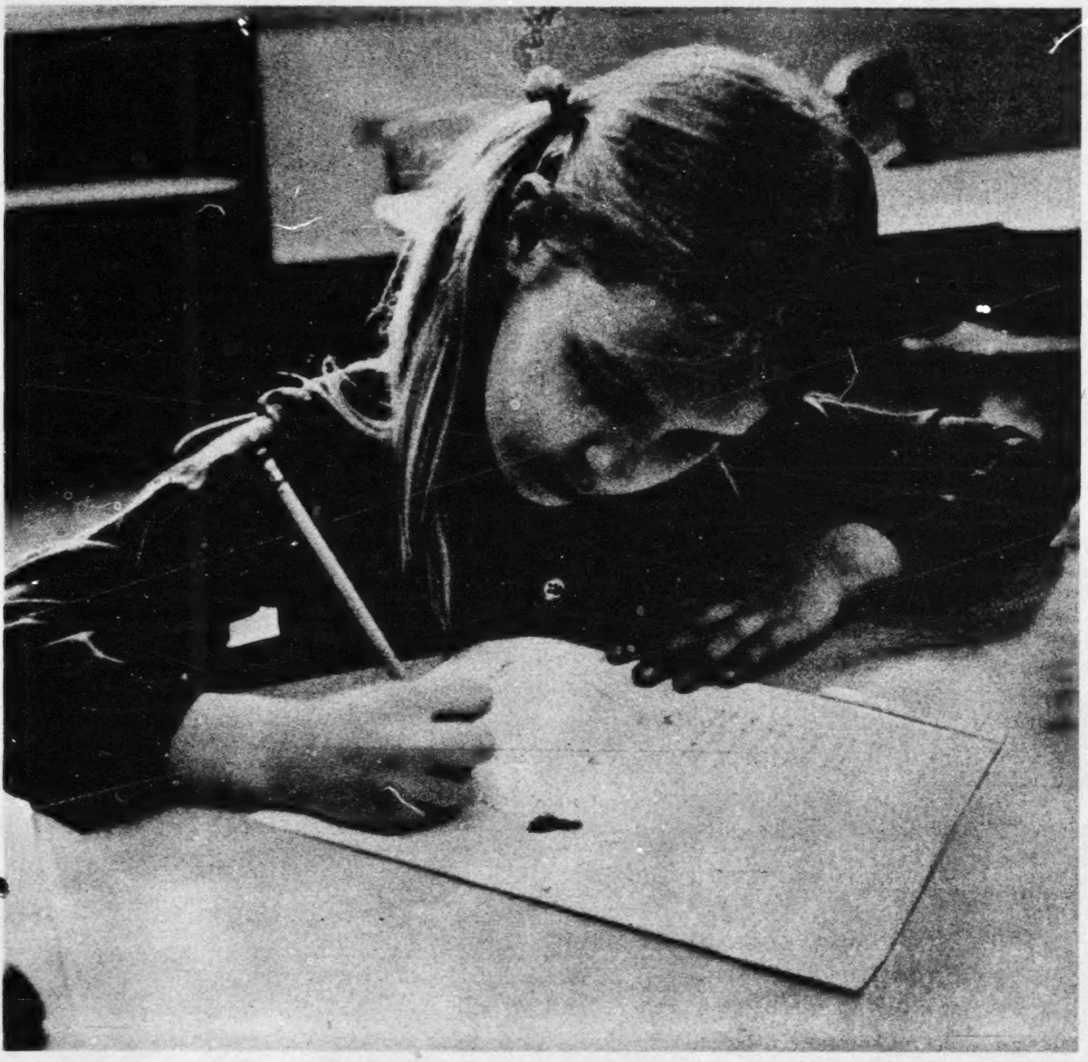
A Cartesian product is another interpretation of multiplication. The various ways of showing Cartesian products give pictorial representations of the multiplication concept. A Cartesian product of two sets is the set of all ordered pairs that can be made by combining each member of the first set with each member of the second set. (This idea is reviewed in the Commentary to Lesson 18.)

Cartesian products are indispensable in planning research projects. No experimental results are valid unless every possible combination of factors is investigated. The use of Cartesian products enables the experimenter to be sure he has not overlooked some combination of factors that must be included in the experimental design.

Lesson 18 reviews the idea of a Cartesian product. The students use multiplication sentences to determine the total number of ordered pairs that can be made from two disjoint sets. You read and discuss a story-problem that motivates the children to determine the total number of combinations of peas and soil that must be tested in an experiment.

Lesson 19 includes another story-problem about testing food for mice. The children find all possible combinations of mice, food and vitamins. On worksheets the students use diagrams, tables and multiplication sentences to determine the Cartesian products of various sets.

Lesson 20 reviews the concepts of arrays and Cartesian products on independent worksheets. The distributive property of multiplication is reinforced.



Lesson 18: REVIEWING CARTESIAN PRODUCTS

Independent worksheets and a story extend the concept of a Cartesian product that was introduced in Unit 25.

MATERIALS

- red, blue, purple and pink crayon for each child
- Worksheets 99 and 100 (independent)
- colored chalk

PROCEDURE

In Activity A the students complete Worksheets 99 and 100 independently or with partners. In Activity B, you read and discuss the story, "Growing Peas."

COMMENTARY

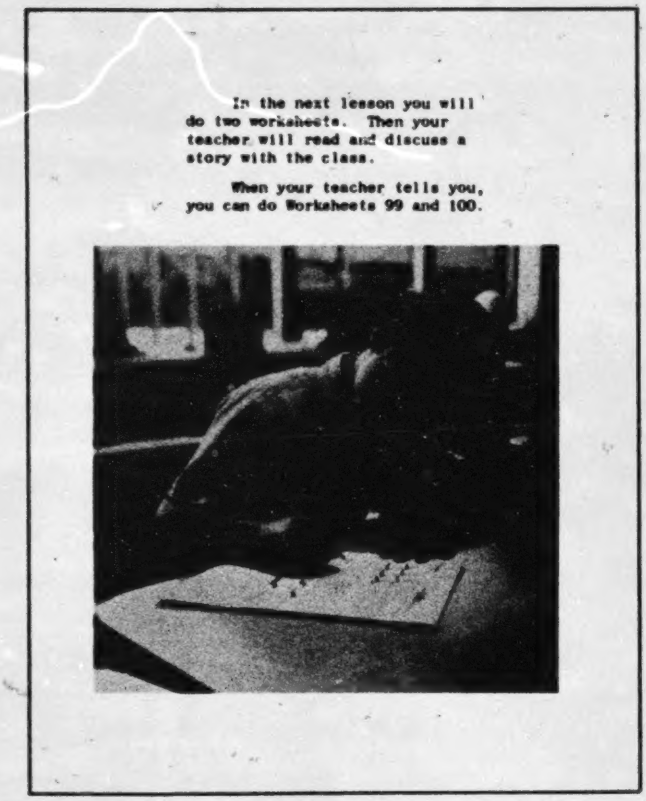
A Cartesian product is another interpretation of multiplication. The various ways of showing Cartesian products give pictorial representations of the multiplication concept. A Cartesian product of two sets is the set of all ordered pairs that can be made by combining each member of the first set with each member of the second set. If the first set is { red, blue } and the second set is { circle, square }, the Cartesian product is { red circle, red square, blue circle, blue square }.

Set A	Set B	Intermediate Step	Pairs in the Cartesian product
red	circle	red < circle square	(red, circle) (red, square)
blue	square	blue < circle square	(blue, circle) (blue, square)

The number of members in the Cartesian product of two disjoint sets (no members in common) is the number of members in the first set multiplied times the number of members in the second set. In the example above there are two members in the first set (red and blue) and two members in the second set (circle and square), so the product is four.

Cartesian products are indispensable in planning research projects. No experimental results are valid unless every possible combination of factors is investigated. The use of Cartesian products enables the experimenter to be sure he has not overlooked some combination.

The words "Cartesian product," are not introduced to the students at this time.



In the next lesson you will do two worksheets. Then your teacher will read and discuss a story with the class.

When your teacher tells you, you can do Worksheets 99 and 100.

Activity A

On Worksheet 99 the students assign pairs of numbers to intersections in a Cartesian coordinate system. They also determine the total number of intersections in the coordinate system by multiplying, for example, the number of streets times the number of avenues.

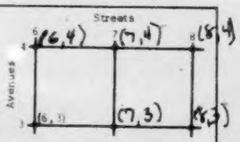
On Worksheet 100 the students work two problems where they can write a multiplication sentence to determine the total number of intersections. On two other problems, they find that the multiplication sentence is not true.

Activity B

Read and discuss the story-problem, "Growing Peas," with the class.

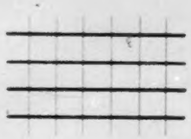
Worksheet 99 Name ____________
Unit 27

1. Write the address at each intersection on this map. (6,3) is already done.



2. There are 3 streets and 2 avenues. There are 3 · 2 or 6 intersections.

3. How many points are there where the thick line segments intersect the thin line segments? 24



4. Did you get 24?
[✓ Yes] No

5. There are 6 thin line segments and 4 thick line segments.

6. There are 6 · 4 or 24 intersections.

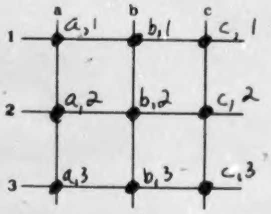
Worksheet 99 (continued)

7. Put a dot at each intersection.

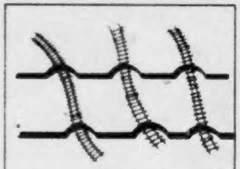
8. Write an ordered pair name at each intersection.

9. There are 3 different letters and 3 different numerals.

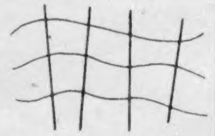
10. There are 3 · 3 or 9 intersections.



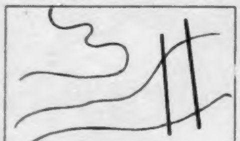
Worksheet 100 Name ____________
Unit 27



There are 2 roads and 3 railroad tracks. There are 2 · 3 or 6 bridges.

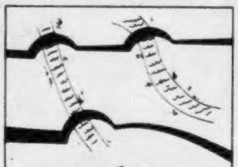


There are 4 straight roads and 3 curved roads. There are 4 · 3 or 12 intersections.



There are 3 curving roads and 2 straight roads. There are 3 · 2 or 6 intersections.

True or [✓ False]



There are 2 streams and 2 roads. There are 2 · 2 or 4 bridges.

True or [✓ False]

Growing Peas

Growing Peas

Farmington is a small town in very good farming country. There is a large canning company there that cans white corn and yellow corn; green beans, yellow beans, lima beans, and kidney beans; and different kinds of peas. People in the canning company test new kinds of vegetables to find better ones to can. Then they provide the seed for these new, better vegetables to the nearby farmers who grow the vegetables that are canned.

Mrs. Blue teaches second grade in Farmington. For three years her classes have had a summer project. They grow vegetables for their families and friends in a big garden just at the edge of town. Some children go every day to take care of the garden.

Last year the corn, beans, tomatoes, beets, carrots, and cabbages were wonderful, but the peas didn't do very well. This year the class wants to change the peas.

"I think we planted the peas in the wrong kind of soil," said Jane.

"No," answered Frank, "I think we used a poor kind of seed. Let's get a new type of pea this year."

Joe said, "Okay, my father can help us decide. He tests seeds for the canning company. I'll ask him what type of peas to plant."

Two days later Joe reported. "My father told me that they have four new kinds of peas that seem to be very good, but he isn't sure which would be best for the class garden. He gave me a package of each kind. Each kind of seed has a name. This one is called Green Monster, this one Baby Wonder, this one Telephone Pole, and this one Early Emerald."

The class was excited about these seeds. "Shall we plant them all?" Sue asked.

"No," Frank answered. "Let's plant only the best."

"But how will we know which seed grows the best kind of plant?" wondered Bill.

After talking about the problem for a while the class decided to test the seeds. There was time to do this before garden planting time.

Jane was still worried about the kind of soil to use. "I don't know what kind of soil is best for pea plants," she said.

"We can try different kinds of soil, also," answered Sam. "Ted and I will go to the garden and bring back samples."

The next day Sam and Ted brought in three pails of soil. There were garden soil, sand, and clay. The class decided they would need to plant each kind of seed in each of the three kinds of soil. If they left out a seed-soil combination it might be the very best one.

Mrs. Blue said, "Here are ten pots left over from last year. Will we need more?"

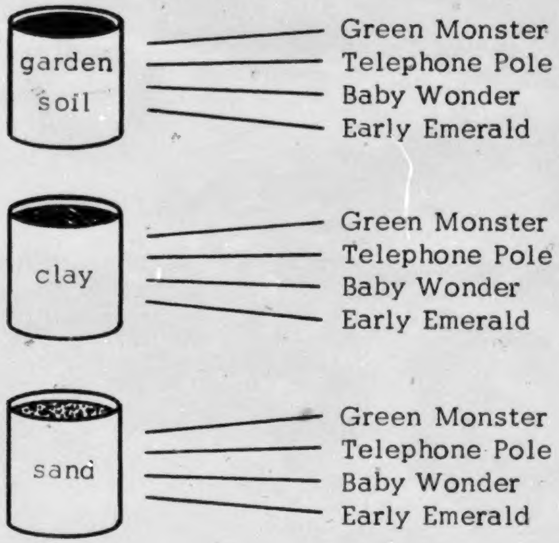
What do you think?

. . . .

Suggest to the children that the combinations be recorded so they can be counted. The children will most likely suggest listing the pairs on the chalkboard, e.g. Baby Wonder in sand. They should discover that there are 12 pairs, but as yet they probably won't notice that:

$$12 = \begin{pmatrix}\text{the number of} \\ \text{kinds of peas}\end{pmatrix} \times \begin{pmatrix}\text{the number of} \\ \text{kinds of soil}\end{pmatrix}$$

Explain that another way to find the combinations is by using a diagram. This is a possibility:



Of course the varieties of peas can be listed first and each associated with three soils. Colored chalk may help clarify the diagrams.

Another useful method is to construct a table. Remind the children of the work they did with Cartesian products in Unit 25. They determined the number of different sundaes they could make with ice cream and toppings. They also determined the number of tests to do with three conditions (moisture, temperature and light) on plants (grass and radishes) and on animals (mealworm beetle, sowbug and earthworm). They made charts for those problems.

	Garden Soil	Sand	Clay
Green Monster	x	x	x
Telephone Pole	x	x	x
Baby Wonder	x	x	x
Early Emerald	x	x	x

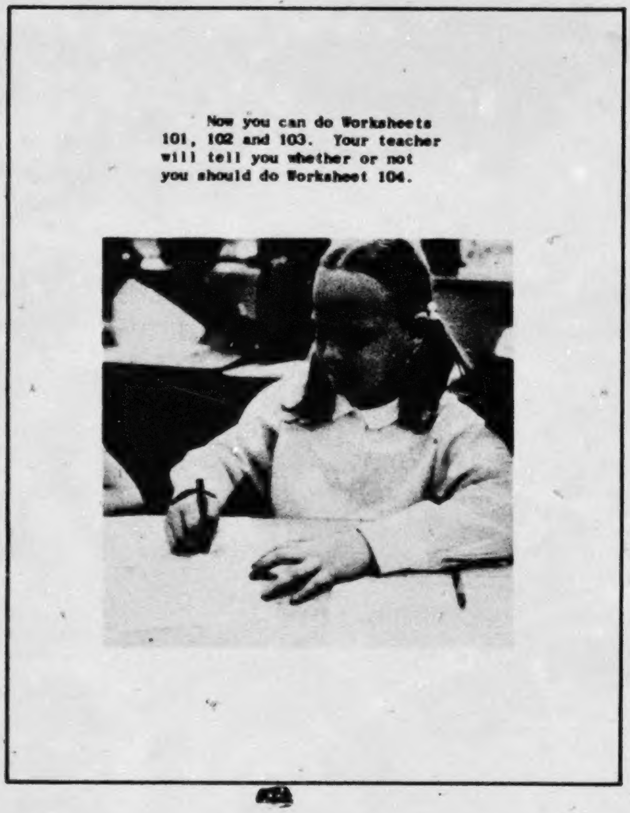
It should be emphasized to the children that in a scientific experiment involving several combinations of components, the result is not valid if a possibility is overlooked. You may also choose to discuss the need for keeping other factors constant. For example, all the pea plants should have as nearly the same amount of light and water as possible.

Lesson 19: MORE CARTESIAN PRODUCTS

More examples of Cartesian products are given in a story-problem and in worksheets. The problems give the students more experience with the multiplicative relation of Cartesian products and with the idea of finding all possible combinations of the members of sets.

MATERIALS

- colored chalk
- Worksheets 101-103 (independent)
- Worksheet 104 (fast workers)



Now you can do Worksheets 101, 102 and 103. Your teacher will tell you whether or not you should do Worksheet 104.

PROCEDURE

In Activity A, you read and discuss a story-problem about testing food for mice. In Activity B the students complete Worksheets 101, 102 and 103 independently or with partners. Fast workers can do Worksheet 104; or, you may want to have all the children do it as part of a class discussion.

Activity A

Read and discuss the story-problem, "Testing Food for Mice."

Testing Food for Mice

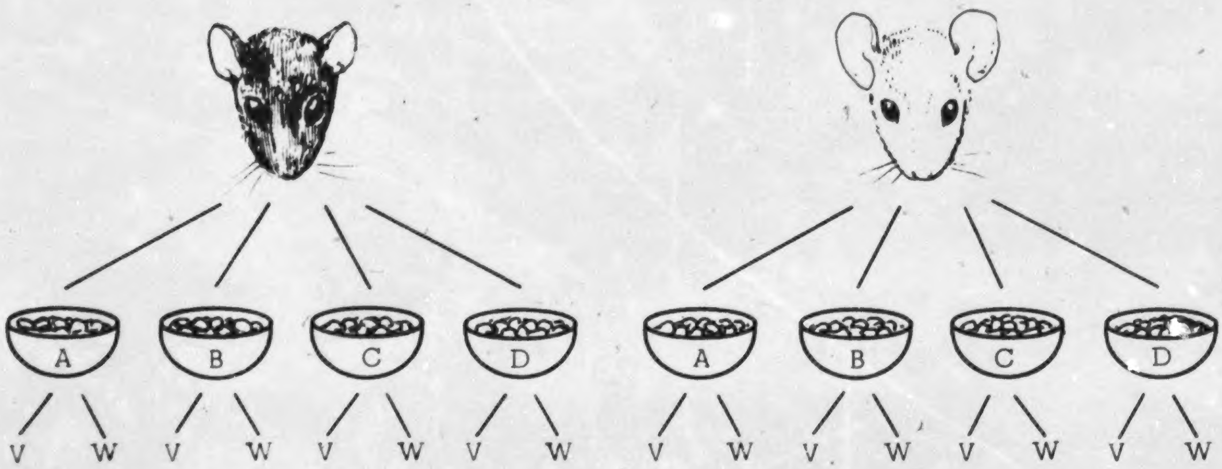
TESTING FOOD FOR MICE

Some men in a big company test food for laboratory mice. They have two kinds of mice: white mice and brown mice. They have four new kinds of food they want to test. So far they don't have names for the kinds of food. They just call them A, B, C, and D. They also want to find out if giving extra vitamins to the mice makes a difference in the way the mice grow. They have two kinds of vitamins: V and W.

How many combinations of mice, food, and vitamins should the men test?

. . . . .

This problem can be diagrammed in various ways. One way is shown here. You could draw the mice for a start. Then children can add to the diagram. Using colored chalk to draw the different kinds of mice, food, and vitamins is effective.



The result of 16 combinations should be obtained by counting. Some children may discover that this is true because:

(number of kinds of mice) x (number of kinds of food) x (number of kinds of vitamins) = (total number of combinations)

$$2 \times 4 \times 2 = 16$$

The formula for obtaining the number of combinations is an excellent discovery. The children should be encouraged to make use of it in finding a Cartesian product.

Activity B

On Worksheet 101 the students determine the number of combinations of dogs and dog food. They do this in three ways: a diagram, a table, and a multiplication sentence. Their answer should be the same with each method.

Worksheet 102 involves the Cartesian product of eye color and wing size of fruit flies. There are two sizes of wings and two colors of eyes. The students write a multiplication sentence to determine that there are four different kinds of fruit flies.

On Worksheet 103 the students determine the Cartesian product of two flower shapes and three colors of African violets.

You may want to discuss Worksheets 102 and 103 with the students at this time.

Worksheet 104 is for fast workers, or can be used with all the students in a class discussion if you want. The students are told to choose one of the three experiments (or to make up their own), and to plan the experiment. They determine the total number of different tests for the experiment. They can make a diagram, make a table, write ordered pairs, or write a multiplication sentence to find the total number of tests. Any students who want to should be encouraged to actually carry out the experiment, or a similar experiment. (See the Minnemast Handbook, Living Things in Field and Classroom.)

Worksheet 101 Name ______________________
Unit 27

Blake's family loves dogs.

They bought 3 new puppies -- a dalmatian, a boxer and a poodle.

The man who sold them the dog suggested 2 different kinds of dog food -- Brand X and Brand Y.

Blake is going to do an experiment to find out which food each dog likes best.

How many experiments will he have to do?

Here are the dogs. Make a diagram to find out how many experiments Blake will have to make.

There will be 6 different experiments.

Worksheet 101 (continued)

Now make a chart to find the number of experiments.

	d	b	p
X	✓	✓	✓
Y	✓	✓	✓

There will be 6 experiments.

Now write a multiplication sentence to find out the total number of different combinations of dogs and dog food.

3 (number of dogs) · 2 (number of foods) = 6 (total number)

There will be 6 experiments.

Did you get the same total in all 3 ways?

Yes

Worksheet 102 Name ______________________
Unit 27

Ed has some fruit flies.

Some of the fruit flies have big wings and the rest have small wings.

Some of the fruit flies have red eyes and the rest have white eyes.

How many sizes of wings are there? 2

How many colors of eyes are there? 2

There are two sizes of wings -- big and small.
There are two colors of eyes -- red and white.

Color the eyes. Some big-winged fruit flies have red eyes and some small-winged fruit flies have red eyes. Some of each have white eyes.

Worksheet 102 (continued)

Make a table to show how many kinds of fruit flies Ed has.

	big wings	small wings
red eyes	✓	✓
white eyes	✓	✓

How many different kinds of fruit flies does Ed have?

Can you write a multiplication sentence to find out? Write it:

2 (number of wing sizes) · 2 (number of eye colors) = 4 (total number of different fruit flies)

Worksheet 103 Name_________________________
Unit 27

Sheldon has some African violet plants.

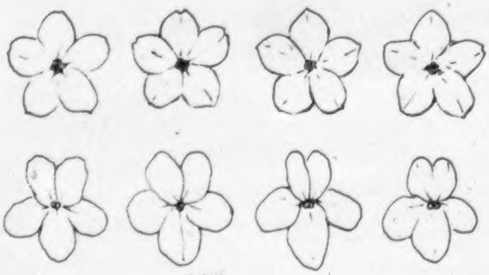
Some plants have star-shaped flowers. The others have irregular-shaped flowers.

Some plants have blue flowers, some have pink flowers, and some have purple flowers.

How many flower shapes are there? 2

How many flower colors are there? 3

Color the flower pictures below to find out how many different kinds of flowers Sheldon has.



Worksheet 103 (continued)

Make a table to show how many kinds of African violets Sheldon has.

	blue	pink	purple
star shape	✓	✓	✓
irregular shape	✓	✓	✓

Write a multiplication sentence to show how many different kinds of flowers there are.

$2 \cdot 3 = 6$

Fast Workers

Worksheet 104 Name_________________________
Unit 27

Pretend you are going to do an experiment. Choose one of these experiments and find out how many different tests you will have to make to complete your experiment.

Find out the number of tests by making a diagram, or making a table, or writing ordered pairs, or writing a multiplication sentence. Use any method you want.

Here are some experiments. Choose one or make up your own.

1. You are testing 2 kinds of fertilizer. The names are Super-Grow and Zoom. You want to test these on 3 kinds of plants -- geraniums, begonias, and coleus. How many tests will you have to make?
2. You are testing 2 kinds of plants -- moss and cactus. You want to test 2 kinds of moisture conditions -- wet and dry. How many tests will you have to make?
3. You are testing 2 kinds of food -- fish food and dry dog food. You are going to test it on 4 kinds of fish -- mollies, guppies, moons and mosquito fish. How many tests will you have to make?

Choose and experiment. Use the next page to plan the different tests you will have to make.

Worksheet 104 (continued)

Answers will vary.

① 2 fertilizers X 3 plants = 6 tests

② 2 plants X 2 conditions = 4 tests

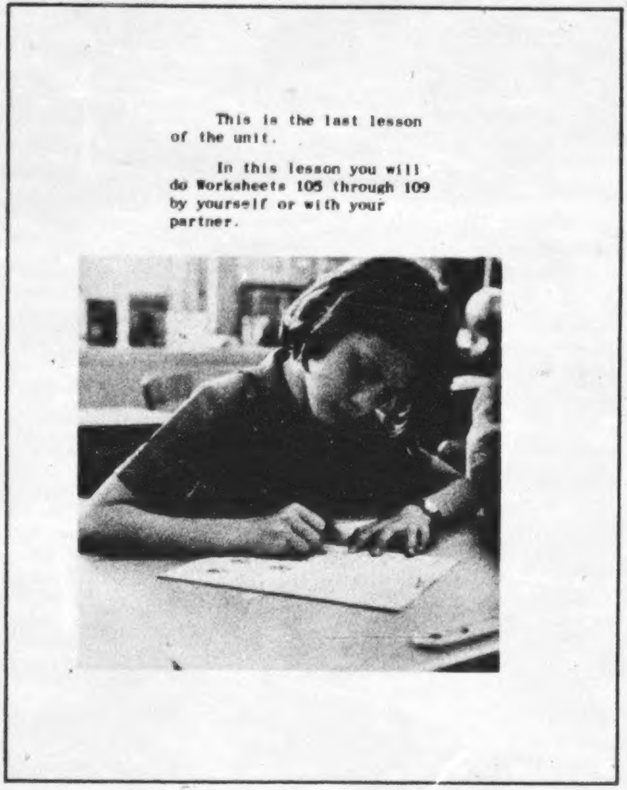
③ 2 foods X 4 fish = 8 tests

For my experiment, I will have to make ______ tests.

Why don't you really try your experiment at school or at home?

Lesson 20: CARTESIAN PRODUCTS AND ARRAYS

This lesson reviews the concepts of arrays and Cartesian products on independent worksheets. The distributive property of multiplication is reinforced. The distributive property: $2 \cdot 10 = (2 \cdot 6) + (2 \cdot 4)$.



This is the last lesson of the unit.

In this lesson you will do Worksheets 105 through 109 by yourself or with your partner.

MATERIALS

- Worksheets 105-109 (independent)

PROCEDURE

Have the children complete Worksheets 105 through 109 independently or with partners.

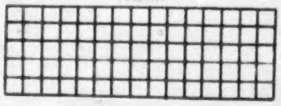
Worksheet 105 reviews partitioning arrays. The students fill in missing parts of multiplication sentences that illustrate the distributive property of multiplication.

On Worksheet 106 the students partition arrays into smaller arrays and complete multiplication sentences that represent the total number of elements in each original array.

On Worksheet 107 the members of a Cartesian product are arranged in a rectangular array. The array is partitioned as an illustration of the distributive property.

On Worksheets 108 and 109 the students determine the total number of ordered pairs that can be written from disjoint sets.

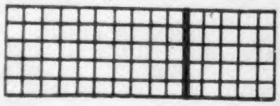
Worksheet 105 Name ___
Unit 27



This array has 5 rows and 15 columns.

There are 5 · 15 or 75 little squares in the array.

Now take the same array as above but partition it into 2 parts.



This part has 5 rows and 10 columns.

It contains 5 · 10 or 50 little squares.

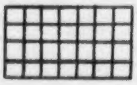
This part has 5 rows and 5 columns.

It contains 5 · 5 or 25 little squares.

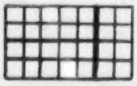
Add the two sections together and you get

(5 · 10) + (5 · 5) = 50 + 25 = 75.

Worksheet 105 (continued)



or



4 rows 7 columns
4 · 7 or 28 squares

4 rows 5 col.
4 · 5 squares

4 rows 2 col.
4 · 2 squares

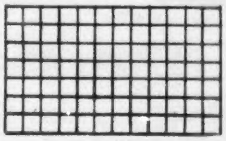
Add (4 · 5) + (4 · 2) = 20 + 8 = 28

Is it true that

4 · 7 = (4 · 5) + (4 · 2) = 28? [✓ YES] NO

Worksheet 106 Name ___
Unit 27

The array to the right has 7 rows and 12 columns.



It contains 7 · 12 or 84 squares.

Answers will vary.

Draw a vertical (↕) line to partition the array into 2 parts.

One part has:
7 rows
___ columns
7 · ___ or 84 squares

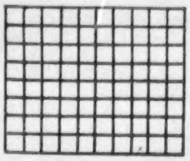
The other part has:
7 rows
___ columns
7 · ___ or 84 squares

Altogether the array has:

(___ · ___) + (___ · ___) = 84 squares

So 7 · 12 = (7 · ___) + (7 · ___) = 84.

Worksheet 106 (continued)



Answers will vary.

The array above has

8 · 10 or 80 squares
rows columns

Draw a vertical line (↕) to partition the array.

PART ONE	PART TWO
8 rows ___ columns	8 rows ___ columns
8 · ___ or ___ squares	8 · ___ or ___ squares

Altogether (8 · ___) + (8 · ___) = 80.

So 8 · 10 = (8 · ___) + (8 · ___) = 80.

Worksheet 107
Unit 27

Name____________________

	red marble	black marble	green marble
Craig	Cr	Cb	Cg
Marvin	Mr	Mb	Mg

Number of boys = 2

Number of colors of marbles = 3

Number of boy-marble ordered pairs = 6

Suppose the boys lose the green marble.

Now how many boy-marble pairs are there?

	red	black	green
Craig	Cr	Cb	Cg
Marvin	Mr	Mb	Mg

n (boys) · n (marbles) =
2 · 2 = 4 boy-marble pairs

n (boys) · n (marbles) =
2 · 1 = 2 boy-marble pairs

Worksheet 107 (continued)

	red	black	green
Craig	Cr	Cb	Cg
Marvin	Mr	Mb	Mg

For 2 boys and 2 marbles, there are 4 boy-marble pairs.

For 2 boys and 1 marble, there are 2 boy-marble pairs.

For 2 boys and 3 marbles, there are 4 + 2 or 6 boy-marble pairs.

You know that 2 · 3 = 6

You also know (2 · 2) + (2 · 1) = 2 · 3

Is this true: 2 · 3 = (2 · 2) + (2 · 1) = 6

[✓ TRUE] FALSE

Worksheet 108
Unit 27

Name____________________

How many different combinations of flowers and vases can you make?

Set A - flowers	Set B - vases
tulips	glass
roses	pottery
zinnias	

Write the ordered pairs. The first one is done.

(tulips, glass)
(tulips, pottery)
(roses, glass)
(roses, pottery)
(zinnias, glass)
(zinnias, pottery)

Fill in this sentence:

3 · 2 = 6

number of members in Set A · number of members in Set B = total number of ordered pairs

---

Worksheet 109
Unit 27

Name____________________

Write the multiplication sentence that tells the total number of ordered pairs.

Set A	Set B	Multiplication Sentence
10 plates	0 spoons	10 · 0 = 0
10 shirts	1 tie	10 · 1 = 10
10 pencils	2 erasers	10 · 2 = 20
10 baseballs	3 bats	10 · 3 = 30
10 fingers	4 rings	10 · 4 = 40
10 telephones	5 people	10 · 5 = 50
10 students	6 teachers	10 · 6 = 60
10 pillows	7 beds	10 · 7 = 70
10 boats	8 lakes	10 · 8 = 80
10 gerbils	9 cages	10 · 9 = 90
10 desks	10 chairs	10 · 10 = 100

Bonus! Make up your own.

10________	11________	10 · 11 = 110
10________	12________	10 · 12 = 120
10________	13________	10 · 13 = 130

MINNEMAST

Job Booklet

Balance Beam System

UNIT 27

Numbers and Their Properties

To the students:

In Unit 24 you performed some experiments and made some discoveries using the Balance Beam System. This booklet will help you further explore the balance beam.

Again you will work with a partner as you perform the tasks of this workbook. Read very carefully before you try to do the jobs or answer the questions. You and your partner should talk over the jobs as you do them, and if you have difficulty or disagree you should go back and read the problem again. Only after you have worked with the problem several times should you ask your teacher for help.

You may work at your own speed but be sure to do your very best. Good luck!

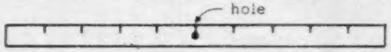
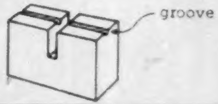
The Authors

Section 1

Job 1

1 Write your partner's name here →

2 Get these materials from your teacher.
One set of materials is enough for both of you.

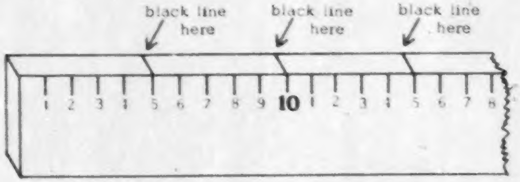
meter stick (called beam)	
wooden block	
8 paper clips	
1 straightened paper clip	
masking tape	

3 Take the beam and a pencil.

Make a black mark on the top edges of the beam at the 5 cm, 10 cm, and 15 cm marks.

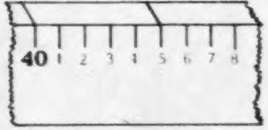
Job 1 continued

4 Does your beam look like this?

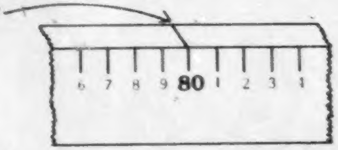


5 Now continue placing marks every 5 cm along the beam: 20 cm, 25 cm, 30 cm, etc.

6 Do you have a mark here?



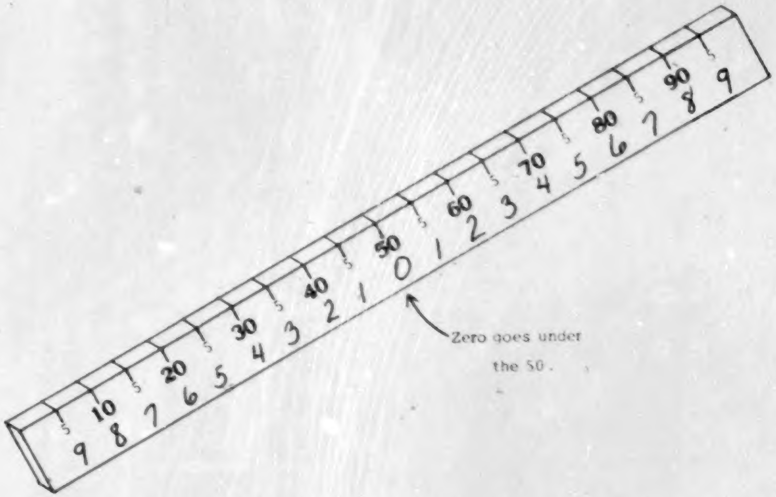
Here?



7 You should have a mark at every numeral that ends in 0 or 5.

Job 1 continued

8 Place numerals on your beam to make it look like this one.

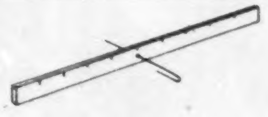


Zero goes under the 50.

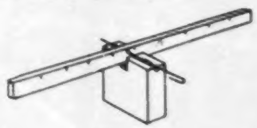
9 Job 1 is over — go on to Job 2.

Job 2

1 Put your balance beam together like this:

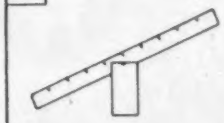


Paper clip goes through hole in beam.

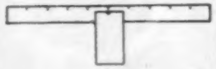


Paper clip rests in groove in block.

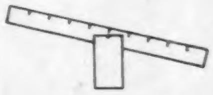
2 Picture ____ describes our balance beam.



not balanced



balanced



not balanced

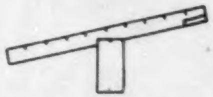
3 The balance beam we made is balanced. True False

Job 2 continued

4 If your beam is balanced go on to Job 3.

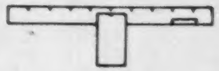
If your beam is not balanced do what the pictures below show you.

1) Tightly stick some masking tape on the arm that is up.



2) Then let go to see if it balances.

3) If the beam balances, go on to Job 3. If it doesn't balance, move the tape back and forth until it balances.

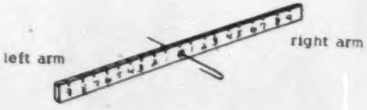


5 Be sure to stick the masking tape on tightly once the beam is balanced.

Job 3

1 Be sure your beam is balanced.

★ In this section we are going to use only the new scale we put on the beam. Ignore all the other scales.



2 Slide a red clip on the left arm of the beam and put it on one of the marks of the new scale.

My red clip is at the ____ mark. Answers will vary.

It is ____ units from the center of the beam.

The center of the beam is the 0 mark.

3 Slide a black clip on the right arm of the beam so that the beam balances.

My black clip is at the ____ mark. Answers will vary.

It is ____ units from the center of the beam.

Job 3 continued

4 Change the positions of the clips. Record the different states in which the beam is balanced.

Answers Will Vary.

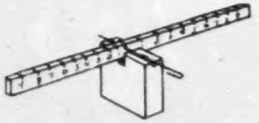
Position of Red Clip	Position of Black Clip
1	1
2	2
3	3
4	4
5	5
etc.	etc.

5 What do you observe about the positions of the red and black clips when the beam is balanced? They are at the same number at each side.

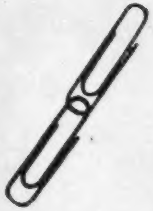
6 Will they always be the same distance from the center of the beam when it is balanced? Yes

Job 4

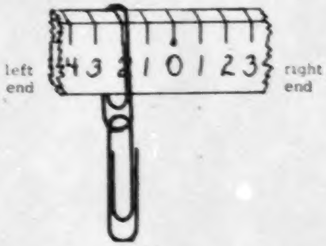
★ Remember — use only the new scale.



1 Put two red paper clips together like this:



2 Place the two clips on the left end of the beam at the 2 mark.



3 Place one black clip on the other end and balance the beam.

The beam balances if the two clips are at the 2 mark and the single clip is at the 4 mark.

Job 4 continued

4 Record this below.

two clips at 2 one clip at 4

(2), (4)

5 Describe each balanced state.

two clips at 3 one at 6 (3), (6)	two at 5 one at 10 (5), (10)
two at 4 one at 8 (4), (8)	two at 6 one at ___ (6), ( )

6 Could you complete them all? NO

Which ones could you not complete? 6, 12

Some children also may not be able to balance the (5,10) state.

Job 5

1 Put one black clip at the 5 mark.

2 Balance the beam by moving the two red clips at the other end of the beam.

3 When the beam is balanced are the two clips on one of the marks of our scale? NO

4 Are they about half way between two marks? Yes

Which two marks? 2 and 3

5 What number do we assign to the point half way between 2 and 3? $2\frac{1}{2}$

6 Record the state of our system using the ordered pair below.

two clips at	one clip at
$2\frac{1}{2}$	5

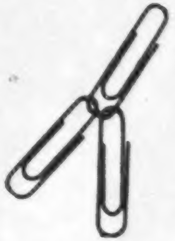
Job 5 continued

7 Use the beam to help complete the ordered pairs.

two clips at:	one clip at:
5	10
3	6
$4\frac{1}{2}$	9
$3\frac{1}{2}$	7
4	8
$1\frac{1}{2}$	3

Job 6

1 Put three clips together like this.



2 Complete these ordered pairs.

(2 , 6)
three clips at, one clip at

3 , 9
three clips at, one clip at

1 , 3
three clips at, one clip at

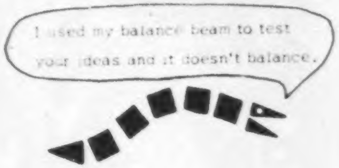
Job 6 continued

3 Put two clips together as you did in Job 5 and complete these ordered pairs.

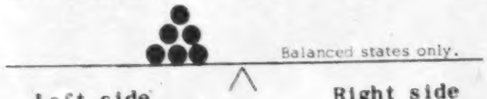
three clips at:	two clips at
2	3
4	6
6	9
3	$4\frac{1}{2}$
5	$7\frac{1}{2}$

Job 7

I think three clips at 4 will balance four clips at 2.


I used my balance beam to test your ideas and it doesn't balance.

1 Using different combinations of clips, experiment and fill in the table.



Left side

Number of clips	Position of clips
3	2
3	4
3	6
3	3
3	Answers will vary
4	3
4	4
4	5
4	4 1/2
4	Answers will vary

Right side

Number of clips	Position of clips
2	3
2	6
2	9
2	4 1/2
2	Answers will vary
2	6
2	8
2	10
2	9
2	Answers will vary

2 Is it always possible to balance the beam? In The examples above, yes. But not in all cases.

3 Place three clips at the 4 mark. Now use one clip on the other arm and balance the beam. Can you do it? NO

4 Do you think you could balance the beam if the arms were longer? YeS

Job 8

1 Use your beam or the table in Job 7 to complete this table. Then fill in the products yourself by multiplying.

Left arm			Right arm		
Number of clips	Position of clips	Product	Number of clips	Position of clips	Product
3	2	6	2	3	6
3	4	12	2	6	12
3	6	18	2	9	18
3	3	9	2	4 1/2	9
3	Answers will vary.		2	Answers will vary.	
4	3	12	2	6	12
4	4	16	2	8	16
4	5	20	2	10	20
4	4 1/2	18	2	9	18
4	Answers will vary.		2	Answers will vary.	

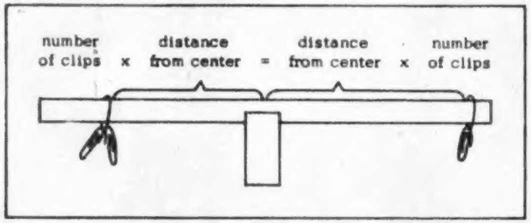
2 From the table and the products you found a rule that will help you balance the beam. Write your rule. The number of clips times the position of clips on the left side equals the number of clips times The position of clips on the right side.

3 Test your rule on the balance beam. Does it work? Yes

Job 9

The Rule of the Balance Beam

The beam balances when:


number of clips x distance from center = distance from center x number of clips

1 Can you use this rule to balance the beam? Yes

2 Try this:

Put 3 clips at 6. Then 3 · 6 = 18.

Where would you put 2 clips so that the beam would balance?

Remember: 2 · 9 must equal 18.

3 If you put 3 clips at 8, then 3 · 8 = 24

If you have 3 clips at 8, where would you put 4 clips to balance the beam?

these must be the same

The 4 clips would be put at 6

because 4 · 6 = 24

Job 9 continued

4 4 clips at 4 will balance 2 clips at 8.

(4 · 4 = 16) (2 · 8 = 16)

these must be the same

5 How many clips would you put at 3 to balance 3 clips at 7?

(7 · 3 = 21) (3 · 7 = 21)

these must be the same

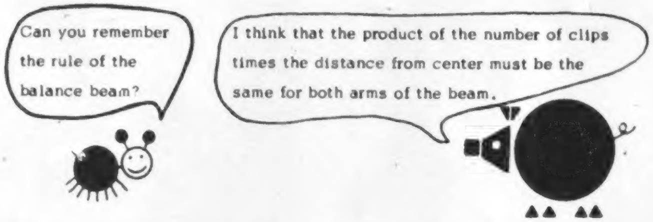
6 Get some clips to test your answers.

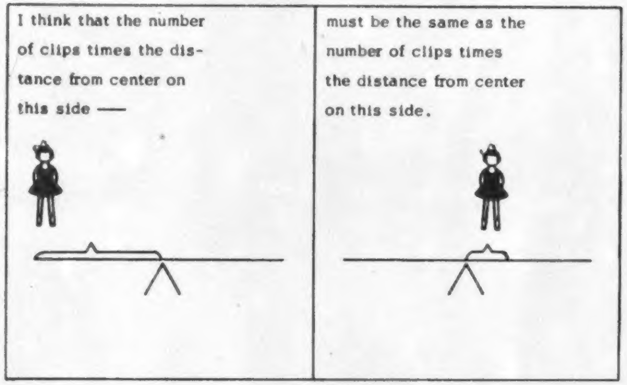
7 Did the rule of the balance beam work for you? Yes

Section 2

Job 10

★ Check to be sure your beam balances without any clips on it. If it doesn't, balance it as you did in Job 2.

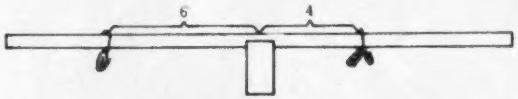

Can you remember the rule of the balance beam?
I think that the product of the number of clips times the distance from center must be the same for both arms of the beam.


I think that the number of clips times the distance from center on this side —
must be the same as the number of clips times the distance from center on this side.

Job 10 continued

So 2 clips at 6 on one side . . . (2 x 6 = 12)

" would balance 3 clips at 4 on the other arm because 3 x 4 = 12.



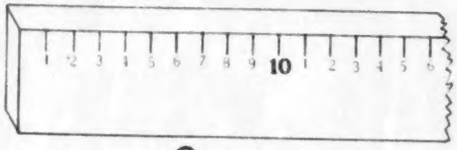
● Do you think that would work if we used the old scale?

When you work the next job you will find out.

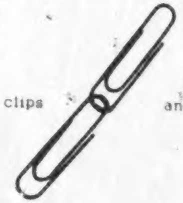
Job 11

We will use the centimeter scale on our beam for the rest of this section.

This is the centimeter scale.



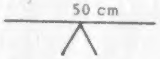
1 Take two red clips



and put them at the 35 cm mark.

How far are these red clips from the center? 15

The center of the
beam is at 50 cm.



To find how far it is from
35 cm to 50 cm subtract

$$\begin{array}{r} 50 \\ -\ \underline{35} \\ ⑮ \end{array}$$

Job 11 continued

2 Now place one black clip on the other end of the beam and balance the beam.

The beam balances when the one clip is at the 80 cm mark.

The one clip is 30 cm from the center.

Remember?

$$\begin{array}{r} 80 \\ -\ \underline{50} \\ ㉚ \end{array}$$

3 Does the rule work?

$$\underset{\text{number of clips}}{\underline{\ 2\ }} \times \underset{\text{distance from center}}{\underline{\ 15\ }} = \underset{\text{number of clips}}{\underline{\ 1\ }} \times \underset{\text{distance from center}}{\underline{\ 30\ }}$$

$$\begin{array}{r} 15 \\ \times\ \underline{2} \\ ㉚ \end{array} \qquad \begin{array}{r} 30 \\ \times\ \underline{1} \\ ㉚ \end{array}$$

2 × 15 does equal 1 × 30.

So the rule does work.

4 Go on to Job 12.

*** If you need practice in subtraction there is a practice sheet after the last job in this booklet.**

Job 12

1 Put two clips at 27 cm.

$$\begin{array}{r} 50 \\ -\,\underline{27} \\ ⑳\!\!\!\!\!\!\;\; \end{array}$$

50
- 27
(23)

The clips are 23 from center

2 × 23 = 46

number of clips × distance from center

2 The beam balances with one clip at the 96 cm mark.

96
- 50
46

The clip is 46 from center

1 × 46 = 46

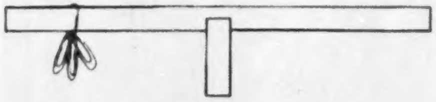
number of clips × distance from center

3 Does the rule of the balance beam work? Yes

✱ **If your products are not equal check your work and balance beam carefully. As a last resort check with your teacher.**

Job 13

1 Use your beam to solve this problem.



3 clips at 82 will balance 4 clips at 26.

3 × 32 = 4 × 24

number of clips × distance from center = number of clips × distance from center

Remember

(82)
- 50
(32)

Remember

50
- 26
(24)

2 Use your beam to solve and check these problems.

2 clips at 41	1 clip at 68	2 clips at 20	3 clips at 70
4 at 43	1 at 78	2 at 16	3 at 74

Subtraction Practice

97 - 33 64	28 - 20 8	55 - 15 40	649 - 421 228
737 - 506 231	21 - 19 2	76 - 49 27	323 - 115 108
517 - 108 309	835 - 219 616	916 - 433 483	347 - 262 85
658 - 170 488	555 - 373 182	1969 - 1492 477	3000 - 987 2013